Managing Capital Outflows: The Role of Foreign Exchange Intervention*

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Abstract

We analyze the optimal intervention policy for an emerging market central bank which wishes to stabilize the exchange rate in response to a capital outflow shock, but possesses limited reserves. Using a stylized framework, we show that the zero lower bound on reserves combined with limited capital mobility generates a time inconsistency problem, and we compare outcomes under full, zero and partial commitment. A central bank with full commitment achieves a gradual exchange rate depreciation to the pure float level by promising a path of sustained intervention, including a commitment to exhaust reserves after particularly adverse shocks. A central bank without commitment intervenes less, wishing instead to preserve at least some reserves forever, and suffers a larger exchange rate depreciation. For more persistent shocks, the time inconsistency problem is larger, and simple intervention rules can achieve welfare gains relative to discretion. We relate the optimal intervention policy to the composition of investors in the FX market.

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1. Introduction

How should central banks in emerging market economies (EMEs) intervene in the foreign exchange (FX) market when faced with capital outflows? As capital flows to EMEs have begun to retrench and reverse after the post-crisis inflows bonanza, and as new risks to the global economy have surfaced, many countries are grappling with this question.

The principle that EME central banks may have reason to undertake sterilized FX intervention in response to inflow shocks has become increasingly accepted. For a start, there is growing recognition that owing to the widespread presence of financial market imperfections, exchange rates can become disconnected from fundamentals and instead turn into a source of shocks (*e.g.*, Gabaix and Maggiori, 2015)[1]. Moreover, several papers have found that sterilized FX intervention has traction on the exchange rate in EMEs, at least under some circumstances (*e.g.*, Blanchard, Adler and Filho, 2015; Chamon, Garcia and Souza, 2015)[2]. As a result, policymakers and academics have conditionally endorsed the use of FX intervention alongside monetary policy in the face of capital inflows (in particular, see Ghosh, Ostry, and Chamon, 2016, and Blanchard, Ostry, Ghosh, and Chamon, 2015). Such research has provided intellectual backing for the growing popularity among EMEs of managed float regimes (as documented by Ghosh, Ostry, and Qureshi, 2015).

However, the optimal FX intervention policy for a managed float regime facing outflow shocks is not well understood. Outflow shocks and inflow shocks are conceptually different, because FX intervention to offset outflow shocks may result in the entire stock of reserves becoming depleted, and optimal policy needs to take this possibility into account. In addition, both the stochastic dynamics of the shock and the categories of investors active in the FX market may differ substantially between outflow and inflow shocks. In the absence of a clear policy framework, policymakers as well as commentators in the financial press have conventionally tended towards recommending no intervention except to counter severe market dysfunction. Indeed, they have often used language associated with the bipolar cases of free floats and pegs—for example, deeming reserves to have been "wasted" if the exchange rate is allowed to move by the end of the outflow episode[3]—and judged it likely that interventions

[1]Gabaix and Maggiori (2015) show how balance sheet constraints on international financial intermediaries cause the exchange rate to become sensitive to financial shocks in addition to the traditional shocks to imports and exports, and they trace the impact of financially-driven changes in the exchange rate onto real variables such as output and unemployment. These authors build on a long literature on exchange rate determination in the presence of financially constrained agents. For example, Jeanne and Rose (2002) show that the existence of noise traders in the FX market means that exchange rate movements may become disconnected from fundamentals. Hau and Rey (2006) connect market incompleteness (specifically, the inability to hedge foreign investments because of a short sale constraint on foreign bonds) to high exchange rate volatility.

[2]Blanchard, Adler and Filho (2015) assess the effects of intervention in response to global inflow shocks. Chamon, Garcia and Souza (2015) analyze the case of Brazil during an outflow episode. For a more extensive literature review, see table 1 of Ghosh, Chamon, and Ostry (2016). Those authors conclude that on the whole, the evidence supports the notion that intervention has some traction on the exchange rate in EMEs.

[3]Wildau and Mitchell (2016) document that "Critics say PBoC [People's Bank of China] spending on

are "counterproductive"[4], implying that they invite speculative attacks and thereby cause an even worse depreciation than would have otherwise materialized.

In practice, EME central banks in managed float regimes have behaved in a heterogeneous manner when faced with capital outflows. We highlight in this paper some notable cases of outflow episodes across different EMEs where sterilized FX intervention was used. We do this not to establish stylized facts for intervention, but rather, to gain some appreciation of the judgments and trade-offs that central banks need to make and that even a simple model should seek to capture[5]. Several salient features stand out: central banks must determine how large their initial reserve stock is relative to the shock; they must assess the probability that the shock continues for many periods; and they must evaluate the composition of participants in the FX market, and judge whether a rapid reserves depletion and/or a sharp exchange rate depreciation is likely to generate a panic.

In this paper, we analyze optimal FX intervention policy in response to capital outflows in a managed float regime. We take explicit account of the zero lower bound on reserves, which is a distinguishing feature of the outflows case, and derive its implications for the optimal policy. We then show how the derived optimal policy depends on the nature of the shock and the composition of participants in the FX market. For purposes of tractability (bearing in mind that we are attempting in this paper to understand the basic fundamentals of the problem), we focus on a stylized theoretical framework where the central bank has an exchange rate target subject to an exchange rate equation, the latter of which nests various forms of limited capital mobility, such as imperfect asset substitutability (*e.g.*, Kouri, 1976, and Blanchard, Giavazzi, and Sa, 2005) and imperfect arbitrage owing to balance sheet constraints on international financial intermediaries (*e.g.*, Gabaix and Maggiori, 2015)[6]. We abstract from alternative policy tools such as interest rates in order to focus on the FX intervention policies[7]. Nevertheless, despite the stylized framework, we believe that our core insights are applicable across a broad range of more elaborate model set-ups.

Our key result is that the zero lower bound (ZLB) on reserves, combined with imperfect capital mobility, generates a time consistency problem which in turn can have a large impact on the optimal policy. Therefore, the central bank's commitment power and communication

intervention has been a waste because it has only delayed further weakness in the renminbi."

[4]Subramanian (2013), when assessing the effectiveness of India's FX intervention during the taper tantrum, states by way of background that "international experience suggests that sterilized intervention to defend a currency, especially during crises, tends to be ineffective or counterproductive."

[5]The specific outflow episodes we highlight are: Russia 2008Q2, Korea 2008Q2, Brazil 2013Q1, India 2013Q2, Russia 2013Q4, and China 2014Q1. See Section 2 for more details.

[6]Gabaix and Maggiori (2015) show that not all forms of limited capital mobility are equal: for there to be a role for FX intervention, it must be that there is a financial imperfection that limits the ability of financial intermediaries to arbitrage excess returns in the FX market.

[7]We do not explicitly consider capital outflow controls, which in our model would simply reduce the magnitude of the outflow shocks. If such controls were perfectly effective, then they would eliminate the need for FX intervention to defend the exchange rate.

strategy are central to the set of implementable policies.

Starting our analysis in the deterministic outflow case (*i.e.*, a constant outflow), we show that a central bank with commitment is able to engineer a gradual exchange rate depreciation to the pure float level during the outflows episode (with the speed of the depreciation being related to the central bank's discount factor). It does so by promising a path of sustained intervention in the future, which is aggressive enough so that the entire reserves stock eventually becomes depleted. The expectation of intervention at a future date within the outflow episode helps stabilize the exchange rate today.

However, the full commitment solution is not time consistent. In the time-consistent solution, the central bank can ignore past promises and simply re-optimize at every date. It intervenes less, both because it wishes to preserve reserves at every date for its own future use in case the shock continues, and also because it recognizes that in the absence of credible promises, the level of the stock of reserves in its vault is the only observable variable that bolsters investors' exchange rate expectations. As a result, irrespective of the history of shocks, reserves never run out. The consequence of lower expected FX intervention throughout the outflow episode is a larger exchange rate depreciation as soon as the outflow episode begins[8].

How can a central bank without full commitment escape the poor exchange rate stabilization outcomes of the time-consistent policy? If the central bank has partial commitment, *i.e.*, it cannot commit to an arbitrarily defined intervention policy but it is able to commit to simple intervention rules (such as a crawling peg or a rule to offset a fixed fraction of any outflow shock), then some escape is possible. Committing to a rule mitigates the large immediate depreciation that is associated with the time-consistent solution, and thereby improves welfare.

Unlike in the alternative bipolar regimes of free floats and pegs, FX intervention and depreciation are jointly optimal under a managed float, even after taking into account that reserves may run out. This result contrasts with many conventional narratives that are skeptical of FX intervention. Our result is, of course, conditional on FX intervention having at least some traction on the exchange rate during the intervention episode.

Having characterized the deterministic outflow case, we turn next to the stochastic outflow case, and we show how the optimal policy responses depend on the assessed persistence of the shock. We show that the persistence of the shock affects the timing of the

[8]The time-consistent solution must be solved using numerical fixed point methods. The solution procedure is not trivial, and the stylized nature of our model makes the problem tractable enough to analyze under a variety of parameter choices. Because of the zero lower bound constraint, we are not able to draw on the numerical solution techniques in the literatures on linear-quadratic, and more general nonlinear, problems (*e.g.*, Levine and Pearlman, 2011; Ambler and Pelgrin, 2010; Blake and Kirsanova, 2011). Adam and Billi (2007) numerically simulate a model with a zero lower bound on monetary policy rates, but they do not face the problem of a non-shock state variable inside the Euler equation. Therefore, we need to implement what is, to the best of our knowledge, a novel approach.

full-commitment FX intervention (which is pushed further into the future for more persistent shocks) and the level of the time-consistent FX intervention (which declines for more persistent shocks). The reason for this comparative static is that the severity of the time consistency problem is related to the shock's assessed persistence. The more persistent is the shock, the longer the outflow episode is expected to last, and the more important are investors' expectations of future interventions in terms of determining the exchange rate today. Therefore, the absence of credible promises to intervene in the future becomes more costly in welfare terms. Simple FX intervention rules dominate the time-consistent solution for more persistent shocks. Conversely, if the outflow shock is known to occur for one period only and disappear thereafter, the optimal full-commitment and time-consistent policies coincide[9].

Finally, we show how the optimal policy responses depend on the composition of investors in the FX market. First, we consider the participation in the FX market of "panickers": a new class of foreign investors who enter the market not based on expected returns, but who sell the domestic currency or reduce their FX lending to domestic banks when they observe central bank reserves being rapidly depleted. If the propensity to panic is high, then intervention has less traction on the exchange rate, and as the central bank undertakes more and more intervention, such actions may become "counterproductive" on the margin—*i.e.*, a further increase in FX intervention may depreciate the exchange rate. The result is poorer exchange rate stabilization even if reserves are plentiful. Second, we discuss "Knightian flight," when a group of investors hold the currency because they (incorrectly) view it as a safe asset, and consequently will sell all their holdings as soon as the exchange rate depreciates. Such investors turn out to hurt welfare under full commitment, but their impact on welfare is ambiguous under time consistency because they provide the central bank with the credibility to implement a temporary peg and prevent a sharp immediate depreciation.

The remainder of the paper is structured as follows. Section 2 highlights some cases where EME central banks have intervened in response to outflow shocks. Section 3 presents our baseline model. Section 4 solves the model for the deterministic case, with subsections on full commitment, time consistency, and simple rules. Section 5 solves the model for the stochastic case, highlighting the role of shock persistence. Section 6 explores how the optimal policy depends on the composition of the FX market. Section 7 concludes.

[9]At a conceptual level, the analysis of the stochastic case highlights why our core insight, regarding the relation between the zero lower bound on reserves and time consistency, is likely to apply more generally to other models in the literature. Suppose that the full-commitment policy for a model without a binding constraint is applied. If the outflow shock process is such that the probability of reserves becoming fully depleted in the future is above zero under this policy, then the mere possibility of the constraint being binding in the future is enough to alter the optimal path today, and the amount by which today's allocation changes depends on investors' expectations regarding future central bank policy.

2. Heterogeneous Responses to Outflow Shocks

Several EME central banks have in fact undertaken sterilized FX intervention to support their currencies in response to capital outflows and/or a sudden decline in inflows. Here we describe some recent cases of outflow episodes. We do not seek to establish a list of stylized facts regarding FX intervention, except for making the observation that central bank behavior is highly heterogeneous and depends on both observable economic variables and the central bank's assessments about the economic environment. Instead, we use a narrative approach to briefly outline the judgments and trade-offs that central banks need to make. We attempt later to capture these judgments and trade-offs in our simple model.

We highlight the following key assessments that the central bank must make:

- The level of reserves relative to the shock's magnitude (which we explore in Section 3).
- The persistence of the shock (which we explore in Section 4).
- The composition of participants in the FX market (which we explore in Section 5).

The following six capital outflow episodes help illustrate the relation between the above assessments and the policies deployed by central bank EMEs:

I. **Russia 2008Q3.** The Central Bank of Russia (CBR) started with a large level of reserves (USD 556bn, or 119 percent of GDP). Faced with a large temporary shock, as the global financial crisis caused a collapse in oil prices and export revenues, the CBR heavily intervened in order to "slow the pace of the rouble's depreciation" and thereby mitigate the "heavy strain on the balance sheets of banks, firms and households via the significant level of foreign-currency-denominated debt that these agents had taken on" (CBR-authored section in BIS, 2013). Reserves fell by USD 187bn over three quarters, while the exchange rate depreciated by 31 percent. Conscious of the possibility that a contraction in banks' external lending might cause further depreciation, the CBR also mitigated outflows by offering unsecured lending to banks.

II. **Korea 2008Q3.** The Bank of Korea (BOK) also started with a large level of reserves (USD 258bn, or 95 percent of GDP) and intervened heavily during the global financial crisis in order to achieve its twin goals: to "contain excessive exchange rate volatility" and to "alleviate the FX funding shortages of banks" (BOK-authored section in BIS, 2013)[10]. The BOK also provided liquidity directly to banks with FX borrowing. Reserves fell by USD 57bn before recovering, while the exchange rate depreciated by 24 percent. The BOK kept FX intervention secret, because they "believe that such information could stimulate speculative trading in the FX market."

[10]Despite Korea's current account surpluses, Korean banks have significant FX borrowing because they are intermediaries for the FX hedging motives of the Korean private sector. For more details, see BIS (2013).

Figure 1: Selected Capital Outflow Episodes in EMEs

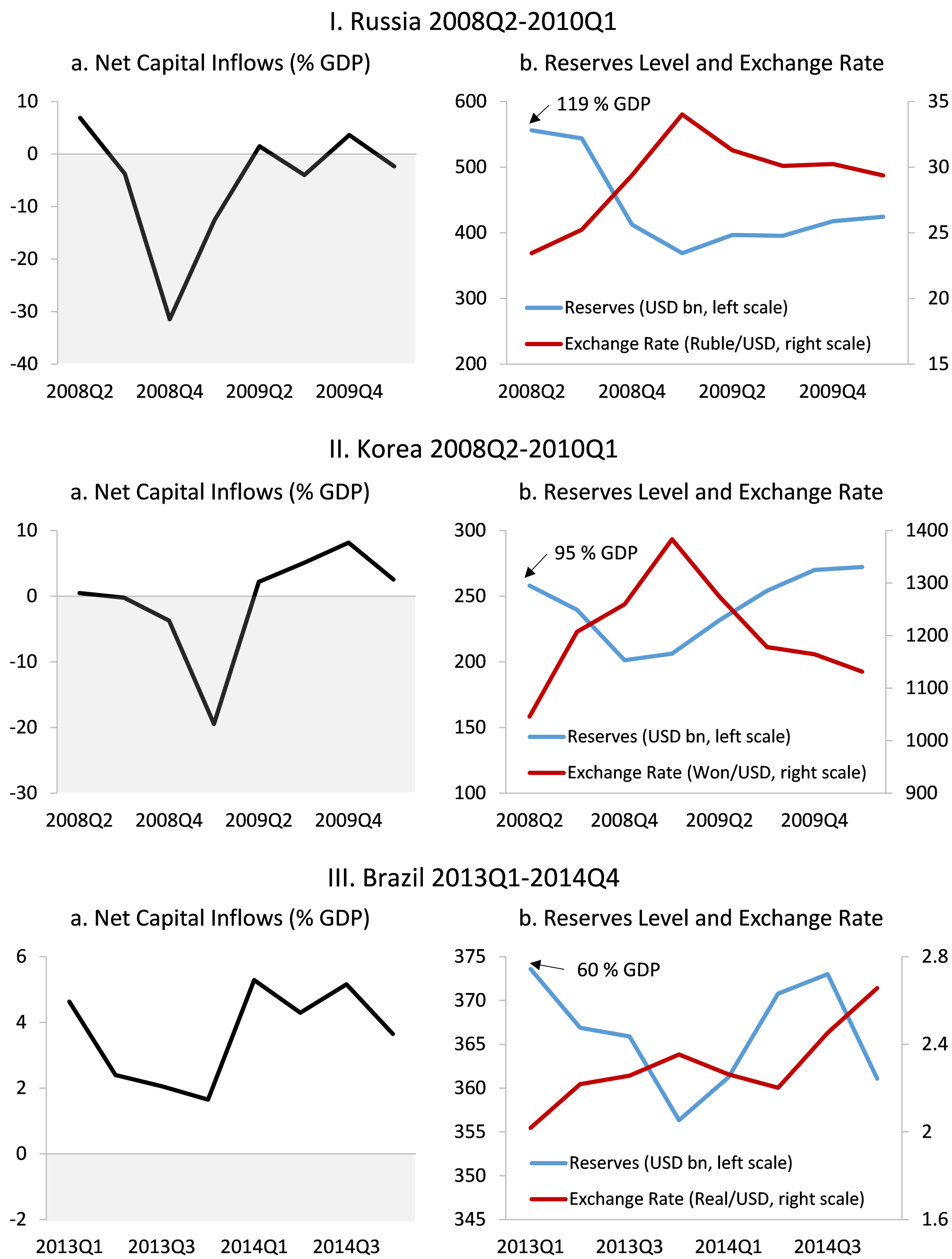
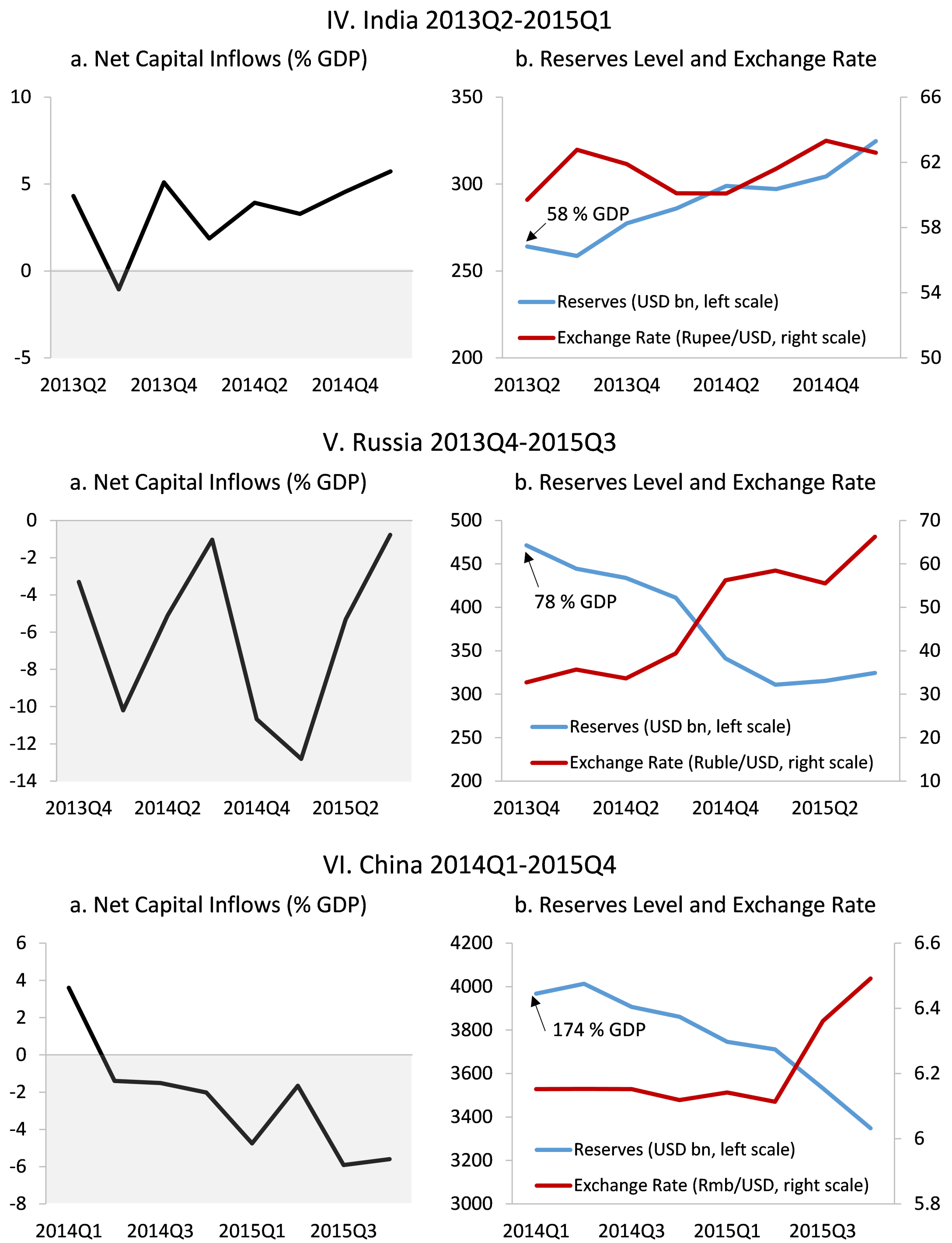


Figure 1 (Continued)



III. **Brazil 2013Q2.** The Brazilian Central Bank (BCB) faced a moderate decline in inflows rather than an outright outflow in 2013, which started at the beginning of the year and was exacerbated by the "taper tantrum" in May. The BCB started with reserves of USD 374bn, or 60 percent of GDP. Following a period of discretionary FX intervention, the BCB decided to announce an intervention rule of daily sales of USD 500m in currency forwards, insuring investors against a domestic currency depreciation, which was reduced in size at the end of the year[11]. Reserves fell by USD 18bn and the exchange rate depreciated by 14 percent.

IV. **India 2013Q3.** India suffered from a reversal in capital flows during the time of the "taper tantrum," which turned out to be moderate and short-lived, but which was seen by some at the time as a harbinger of future trends as advanced economies began to normalize monetary policies. The Reserve Bank of India's (RBI) moderate reserves were large relative to the immediate shock (USD 264bn, or 58 percent of GDP), although not to a sustained continuation of outflows. The RBI intervened by lending in USD to state-owned oil companies (Subramanian, 2013), and later allowing FX losses by the companies to be repaid in rupees instead of USD (Indian Express, 2014). The intervention was small, and reserves fell by just USD 5bn; the exchange rate depreciated by 5 percent.

V. **Russia 2014Q1 and Q4.** Russia was hit by a sequence of two outflow shocks in 2014, the first as a result of the beginning of the military intervention in Ukraine, and the second later in the year owing to Western sanctions and the collapse in oil prices. Relative to the 2008 crisis, the CBR started with a lower level of reserves (USD 471bn, or 78 percent of GDP), and the shock was smaller (albeit still large) and more permanent. Reserves fell by USD 160bn over five quarters (so intervention was smaller but more sustained than in the 2008 crisis), while the exchange rate depreciated by 44 percent over five quarters and continued depreciating after the intervention had been stopped. The CBR also provided capital support to banks to ease their FX deleveraging process (IMF, 2015).

VI. **China 2014Q2.** The People's Bank of China (PBC) started with the largest level of reserves of all the EME examples considered here (USD 3.97tn, or 174 percent of GDP). As the Chinese economy weakened in 2014, capital outflows picked up, and then worsened in mid-2015. The persistence of the shock remained unclear. During this period, China was moving to a managed float regime from a fixed peg. The PBC used its warchest of reserves to maintain the exchange rate almost unchanged for five quarters before allowing some depreciation. While some observers deemed the

[11] Using a variety of approaches, Chamon, Garcia and Souza (2015) argue that the announcement of the new intervention rule was effective in mitigating the depreciation of the Brazilian real.

> interventions to have been "wasted" because the exchange rate eventually moved, some PBC officials were reportedly cautiously pleased that they had managed to contain some of the depreciation pressures, because a sharp depreciation carried the risk of generating a larger panic: "Once confidence is lost, it can't easily be restored" (Wildau and Mitchell, 2016)[12].

We now turn to constructing a model which can help shed light on optimal policy as a function of the level of reserves, the nature of the shock, and the composition of the FX market. We begin with a simple baseline model and then extend it in several steps to shed light on several of the practical considerations which emerge from the above narratives.

3. Stylized Model

Our starting point is a stylized model of exchange rate determination motivated by imperfect arbitrage between domestic and foreign assets. The capital flow equation is as follows:

$$k_t\left(s^t\right) = a\left(E_{s^t} e_{t+1}\left(s^{t+1}\right) - e_t\left(s^t\right)\right) + z_t\left(s^t\right), \tag{1}$$

where k_t represents capital outflows, e_t is the exchange rate (defined so that an increase means a depreciation), and z_t denotes a capital outflow shock. s_t is the state of nature in period $t = 0, 1, 2, ...$ and $s^t \equiv \{s_0, s_1, s_2, ..., s_t\}$ is the history of shocks up to period t. $E_{s^t} e_{t+1}\left(s^{t+1}\right) \equiv \sum_{s^{t+1} \succ s^t} e_{t+1}\left(s^{t+1}\right)$ is the expected exchange rate for period $t+1$ over the histories s^{t+1} which are feasible given the history s^t up to period t.

A finite value for a reflects a limit to arbitrage by the private sector, which in turn creates an opening for welfare-improving central bank policies. By contrast, as the parameter $a \to \infty$, the equation above tends to the standard perfect-arbitrage uncovered interest parity condition. Our specification is conceptually related to the framework in Gabaix and Maggiori (2015), where there is a limit to the arbitrage between domestic and foreign assets because the financial intermediaries who must conduct such arbitrage face balance sheet constraints[13]. Notice also that the portfolio balance models of Kouri (1976) and Blanchard, Giavazzi, and

[12] Wildau and Mitchell (2016) document the anonymous PBC official's comments as follows: "The cost of intervention in terms of reserves has been high but this policy can't be evaluated just in terms of numbers. Once confidence is lost, it can't easily be restored. Then a lot of bad things can happen."

[13] Our parameter a can be compared with the variable $\frac{1}{\Gamma}$ in Gabaix and Maggiori's (2015) framework, where Γ is related to the portion of shareholders' funds that financial intermediaries are able to steal, and therefore measures the strength of financial frictions for investors. Under this interpretation, as $a \to \infty$, $\Gamma \to 0$ and financial frictions disappear, so perfect arbitrage becomes possible. In our model, however, unlike in Gabaix and Maggiori's setup, tomorrow's exchange rate is not assumed to be determined by the unwinding of carry trades undertaken today. The reason is that we imagine a large number of different agents undertaking new carry trades in every period, with the net capital outflow being determined by the expected depreciation between today and tomorrow rather than by past transactions.

Sa (2005) produce exchange rate equations which are broadly consistent with our above equation. We implicitly assume that the domestic and foreign interest rates are identical or that the wedge between them is constant (and therefore absorbed into the shock $z_t\left(s^t\right)$), so that the policy rate is not a separate item in the toolkit[14].

We impose a simple linear formulation for the current account surplus, which is normalized so that the current account is in balance when $e_t\left(s^t\right) = 0$:

$$ca_t\left(s^t\right) = ce_t\left(s^t\right). \tag{2}$$

Finally, the central bank's policy variable is the level of sterilized FX intervention $f_t\left(s^t\right)$:

$$f_t\left(s^t\right) \equiv R_t\left(s^{t-1}\right) - R_{t+1}\left(s^t\right) \text{ subject to } R_{t+1}\left(s^t\right) \geq 0 \text{ and } f_t\left(s^t\right) \geq 0, \tag{3}$$

where $R_t\left(s^{t-1}\right)$ is the stock of reserves that is available at the beginning of time t and is determined by the FX intervention policies up to time $t-1$, and R_0 is the exogenous level of initial reserves. The first constraint is the zero lower bound (ZLB) on reserves, so the central bank is conscious of the possibility that reserves may run out. Since we are focused on an outflow episode, we also impose (for analytical convenience) that reserve accumulation is not possible, but our core results are robust to the relaxation of this assumption.

The balance of payments identity is as follows:

$$k_t\left(s^t\right) \equiv ca_t\left(s^t\right) + f_t\left(s^t\right). \tag{4}$$

Substituting equations (1), (2), and (3) into the identity (4), we derive the reduced set of equations that fully characterizes the feasible set of the model.

Definition 1 (Reduced-form model) *The reduced-form version of the model is described by the equations for the exchange rate and FX intervention:*

$$e_t\left(s^t\right) = \frac{1}{a+c}\left(z_t\left(s^t\right) - f_t\left(s^t\right) + aE_{s^t}e_{t+1}\left(s^{t+1}\right)\right) \tag{5}$$

$$f_t\left(s^t\right) = R_t\left(s^{t-1}\right) - R_{t+1}\left(s^t\right) \in \left[0, R_t\left(s^{t-1}\right)\right]. \tag{6}$$

The exchange rate equation can be iterated forward to yield:

$$e_t\left(s^t\right) = \frac{1}{a+c}E_{s^t}\sum_{i=0}^{\infty}\left(\frac{a}{a+c}\right)^i\left[z_{t+i}\left(s^{t+i}\right) - f_{t+i}\left(s^{t+i}\right)\right]. \tag{7}$$

[14]Ghosh, Ostry and Chamon (2016) show in a model without a lower bound on reserves that if the policy rate is available, it should be used alongside FX intervention so as to stabilize the exchange rate (*i.e.*, higher interest rate after outflow shocks). We abstract from such considerations in this paper and focus instead on the simplest model with a zero lower bound on reserves.

Therefore, in our model, FX intervention affects the exchange rate. This result is consistent with the empirical evidence for EMEs in Blanchard, Adler, and Filho (2015). Our model includes an expectations channel: FX intervention to support the exchange rate in future periods supports the exchange rate today as well. One unit of FX intervention today appreciates the exchange rate by $\frac{1}{a+c}$ today, while one unit of FX intervention tomorrow appreciates the exchange rate today by the lower amount $\frac{a}{(a+c)^2}$. We denote the pure float exchange rate level $\overline{e}\left(s^t\right)$ as the level of the exchange rate in the absence of any intervention at all.

In this paper, we focus on two different stochastic processes for outflow shocks.

Definition 2 (Capital outflow shock) *The outflow shocks $\left\{z_t\left(s^t\right)\right\}_{t=0}^{\infty}$ are assumed to have one of the following structures:*

- *Deterministic constant: $z_t\left(s^t\right) = \overline{z} > 0$ for all t and s^t.*
- *Finite markov with absorbing state: $\left\{z_t\left(s^t\right)\right\}_{t=0}^{\infty}$ evolves according to following Markov transition matrix, where the columns represent $z_t\left(s^t\right)$ and the rows represent $z_{t+1}\left(s^{t+1}\right)$:*

$$\begin{array}{cc} & \begin{array}{cc} \overline{z} & 0 \end{array} \\ \begin{array}{c} \overline{z} \\ 0 \end{array} & \left[\begin{array}{cc} p & 1-p \\ 0 & 1 \end{array}\right] \end{array}, \quad \textit{where } z_0\left(s^0\right) = \overline{z} > 0. \tag{8}$$

The deterministic shock involves a constant outflow forever, which means that irrespective of the initial level of reserves, it is impossible to offset the entire shock. Section 4 is devoted to the case of deterministic outflows, and characterizes the full-commitment solution, the time-consistent solution, and the solutions under simple FX intervention rules. The stochastic shock begins at an outflow level of $\overline{z}$ and then in every period, it has a probability p of persisting at the same level into the next period, and a probability $1-p$ of falling to zero and remaining there forever. Irrespective of the initial level of reserves, the probability that FX intervention can fully offset the entire sequence of capital outflow shocks is less than one. Section 5 focuses on the stochastic outflows case.

Finally, the central bank's objective function shows a preference for stabilization.

Definition 3 (Welfare objectives) *Given a discount factor $\beta \in [0,1)$, a given state-contingent sequence of FX intervention $\left\{f_t\left(s^t\right)\right\}_{t=0}^{\infty}$ is evaluated in terms of the deviations of the sequence of exchange rates $\left\{e_t\left(s^t\right)\right\}_{t=0}^{\infty}$ from a given target $e^\star$:*

$$W(R_0, \left\{f_t\left(s^t\right)\right\}_{t=0}^{\infty}) = -E_{s_0}\left[\sum_{t=0}^{\infty}\beta^t \frac{\left(e_t\left(s^t\right) - e^\star\right)^2}{2}\right]. \tag{9}$$

$e^* \neq \overline{e}\left(s^t\right)$ is the exchange rate target relevant for the episode. This specification captures in reduced form the notion that exchange rate fluctuations can be destabilizing under certain conditions, for example by causing adverse shifts in the terms of trade for households, or by tightening the borrowing constraints of domestic agents who borrow in foreign currency (*e.g.*, Aghion, Bacchetta, and Banerjee, 2001; Mendoza, 2002)[15].

Notice that our model is designed for EMEs with managed floats, and e^* does not represent an exchange rate peg: there is no obligation that the exchange rate be maintained at that level as long as possible. When faced with a permanent shock, e^* represents the initial pre-shock exchange rate which the central bank wishes to minimize deviations from, but knows is not feasible in the long run. When faced with a temporary shock, e^* represents both the initial pre-shock exchange rate and the long-run exchange rate, but the shock may temporarily cause the actual exchange rate to depreciate above e^*.

We conclude this section with a note on the generality of the results that can be derived from this model. On the one hand, our optimal solutions for the exchange rate path and FX intervention level will clearly depend on the functional forms for welfare and the exchange rate equation outlined above, and on our decision to ignore FX accumulation. Nevertheless, the qualitative effect of the ZLB on the time consistency of the solution, and on the comparative levels of FX intervention and welfare across different degrees of central bank commitment, will robustly apply across a wide range of models, as long as in those models, there exists a history $\{s^t | \Pr(s^t) > 0\}$ at which reserves would run out if the central bank followed the unconstrained full-commitment policy.

4. The ZLB on Reserves and Time Consistency

In this section, we prove that the ZLB on reserves, combined with imperfect capital mobility, generates a time consistency problem which in turn can have a large impact on the optimal policy. In practical terms, the implication is as follows. For very large levels of reserves, the optimal FX intervention policy does not depend much on the degree of commitment of the central bank, and takes the form of fully offsetting the outflow shock and keeping the exchange rate stable. For low to moderate levels of reserves, the optimal FX intervention policy depends on the central bank's commitment power. If the central bank has high commitment power (perhaps owing to an already-existing and clearly-communicated FX intervention strategy, or an extensive history of prior interventions), it can engineer a gradual depreciation to the pure-float level (as many EME central banks in section 2 attempted to achieve). In the absence of commitment power, the central bank may be compelled to un-

[15]In a standard New Keynesian (NK) model with imperfect financial markets, monopolistic competition, and home bias, a squared quadratic term for the exchange rate around its steady state level will naturally be a part of the central bank's objective function, when the policy rate is not included as an instrument in the policymaker's toolkit (see, for example, Cavallino, 2015). More broadly, our objective function is intended to also capture in reduced form a range of costs stemming from balance sheet effects that have not yet been fully captured in standard NK treatments, but which are a key worry for EME policymakers.

dertake only small FX intervention and to let the exchange rate depreciate; and surprisingly, the ability of the central bank to stabilize the exchange rate is worse for more patient central banks. A central bank with some commitment power, but with less of a history in undertaking heavy interventions and/or facing investors with less anchored expectations, would find it optimal to announce a simple FX intervention rule which involves sustained future intervention (as in the Brazilian outflow episode in section 2).

Throughout this section, we focus on the deterministic capital outflow shock described in definition 2, where $z_t(s^t) = \overline{z} > 0$ for all t and s^t. The pure-float exchange rate is now $\overline{e}(s^t) = \overline{e} = \frac{z}{c}$. The value of every variable depends only on time t and not otherwise on the state of nature s^t, so the path of every variable $\{x_t(s^t)\}_{t=0}^{\infty}$ can now be written simply as $\{x_t\}_{t=0}^{\infty}$. Expectation terms are no longer needed, because there is no uncertainty, so $E_{s^t} x_{t+1}(s^{t+1})$ can be written simply as x_{t+1}. We return to the stochastic outflow shock in the next section. The proofs for all the results in this paper are relegated to the Appendix, subsection 8.1.

4.1. Constant outflows $\overline{z}$ under full commitment

A central bank with full commitment credibly commits in period $t = 0$ to the entire future FX intervention path $\{f_t\}_{t=0}^{\infty}$. This promised FX intervention policy pins down the path of exchange rates $\{e_t\}_{t=0}^{\infty}$. All policy promises are feasible and unbreakable, so foreign investors expect central banks to keep their word. Of course, full commitment is not a realistic assumption, but it establishes a benchmark for the second-best policy and welfare level[16].

Definition 4 (Commitment problem) *The commitment solution comprises paths of FX intervention $\{f_t\}_{t=0}^{\infty}$, exchange rates $\{e_t\}_{t=0}^{\infty}$ and reserves stocks $\{R_{t+1}\}_{t=0}^{\infty}$ which solve:*

$$W^{FC}(R_0) = \max_{\{e_t, f_t\}_{t=0}^{\infty}} -\sum_{t=0}^{\infty} \beta^t \frac{(e_t - e^\star)^2}{2} \tag{10}$$

subject to, for each period t:

$$\begin{aligned}
\Gamma_t : \quad & e_t = \frac{1}{a+c}\left[\overline{z} - f_t + a e_{t+1}\right] \\
\Pi_t : \quad & R_t - R_{t+1} = f_t \\
\Psi_t : \quad & f_t \geq 0 \\
\Phi_t : \quad & R_t \geq f_t
\end{aligned}$$

[16]The maximum welfare achievable through the use of FX intervention will be below the first-best welfare level because reserves are by nature a non-contingent instrument, and therefore are welfare-inferior to the first-best instrument of history-contingent insurance.

$$\Lambda: \quad R_0 \geq \sum_{t=0}^{\infty} f_t,$$

where the respective multipliers on the (in)equalities are Γ_t, Π_t, Ψ_t, Φ_t, and Λ. The initial level of reserves R_0 is given exogenously.

Notice that in each period t, the exchange rate e_t is affected by the FX intervention in the same period f_t and by the next period's exchange rate e_{t+1}. The latter variable is a sufficient statistic capturing the effect of the entire promised path of future FX interventions $\{f_{t+i}\}_{i=1}^{\infty}$ on e_t. Therefore, the commitment problem can be written using the Bellman formulation.

Lemma 1 (Bellman representation) *The commitment solution solves:*

$$v^{FC}(R,\mu) = \max_{e,R',\mu'} \left\{ -\frac{(e-e^*)^2}{2} + \beta v^{FC}(R-f,\mu') \right\}$$

subject to:

$$\begin{aligned}
\gamma: \quad & e = \frac{1}{a+c}\left[\bar{z} - f + a\mu'\right] \\
\pi: \quad & R - R' = f \\
\psi: \quad & f \geq 0 \\
\phi: \quad & R \geq f \\
\delta: \quad & \mu = e \\
\sigma: \quad & \mu' \geq \bar{e} - \frac{1}{a+c}R',
\end{aligned}$$

where the respective multipliers on the (in)equalities are γ, π, ψ, ϕ, δ, and σ, and where the final constraint is derived recursively from the definition of the feasible set $M(R)$:

$$M(R) = \left\{ \mu : \mu = \frac{1}{a+c}\left(\bar{z} - f + a\mu'\right) \text{ for some } \mu' \in M(R') \text{ and } f = R - R' \in [0,R] \right\}.$$

Therefore, the infinite-horizon problem can be broken down into a sequence of two-period problems, in each of which the central bank has two policy instruments: today's FX intervention f (which must validate the promise μ for today's exchange rate that has been inherited from the past[17]) and today's promise for tomorrow's exchange rate μ'. Because of ZLB on reserves, the lowest exchange rate μ' that can be promised for the next period is related to the level of reserves R' that is left for the next period[18]. We need to make sure that the optimal policy derived never violates this condition.

[17] The variable μ is a pseudo-state variable, in the terminology of Kydland and Prescott (1980), and we need to add the separate "promise-keeping" constraint, $\mu = e$, to keep track of it.

[18] It turns out that the most appreciated exchange rate that is feasible is achieved by spending the entire stock of reserves immediately. This result will be important for the time-consistent case.

In what follows, we stay within the notation of the infinite-horizon problem (we return to the Bellman representation again in subsection 4.2, because the time consistency problem can be solved only by recursive methods). We first derive the marginal value of FX intervention for any period t.

Lemma 2 (Marginal value of intervention) *The marginal value of FX intervention at date t on welfare at date 0 is:*

$$\Gamma_t = \sum_{u=0}^{t} \beta^u \left(\frac{a}{a+c}\right)^{t-u} (e_u - e^*) \,. \tag{11}$$

The marginal effect on date 0 welfare of FX intervention in period t is positive in every period t, and its time path depends on the interplay between opposing forces. On the one hand, the promise of FX intervention in any period t appreciates exchange rates in all prior periods. This effect grows as t increases, causing the marginal value of intervention evaluated at date 0 to increase over time. On the other hand, the central bank's welfare criterion discounts the future, and intervention in period t has a stronger effect on the exchange rate in periods $u \leq t$ than does intervention at future dates. These factors tend to reduce the marginal value of intervention over time.

Lemma 3 (No intervention case) *When $\{f_t\}_{t=0}^{\infty} = \{0\}$, the marginal value of intervention is hump-shaped with one maximum in period $t^* = \arg\max_t \{\Gamma_t\} > 0$.*

Lemma 4 (Small initial reserves) *If the initial level of reserves is very small, then it is optimal to use all reserves in period t^*.*

Figure 2: Full Commitment, Small Initial Reserves $R = 0.1$

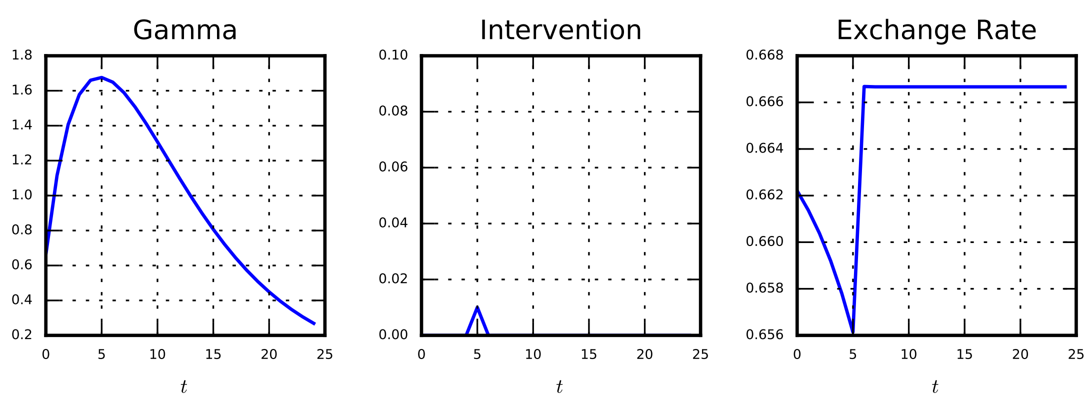


Figure 2 illustrates both the above results using baseline parameters such that $\frac{1}{\beta} = \frac{a+c}{c}$, $e^* = 0$, and $\bar{z} = 0.1$. Because of the hump-shaped graph for Γ_t, even though the outflow episode begins at $t = 0$, the optimal strategy of the central bank is not to intervene at $t = 0$, but at some point in the future $t^* > 0$. The promised future intervention causes all prior exchange rates to appreciate slightly below the pure float level $\bar{e}$.

Using the principle that reserves are optimally used in periods when the marginal value of intervention is the highest, we can now derive the general solution.

Proposition 1 (Euler equation) *At any date t such that f_t is in the interior of its feasible set (i.e., intervention is used and reserves have not run out yet):*

$$\Gamma_t = (a + c)\,\Lambda \tag{12}$$

$$(e_t - e^*) = \beta\,(e_{t+1} - e^*)\,. \tag{13}$$

Theorem 1 (Commitment solution) *For a positive level of initial reserves R_0, there exists a non-empty subset of consecutive periods $[t_1, T] \subset [0, \infty)$ at which FX intervention is optimal: FX intervention is zero for $t < t_1$, positive for $t \in [t_1, T]$, and zero for $t > T$, since reserves run out in period T. The exchange rate follows the path:*

$$e_t = \begin{cases} \bar{e}\left(1 - \left(\frac{a}{a+c}\right)^{t_1 - t}\right) + \left(\frac{a}{a+c}\right)^{t_1 - t} e_{t_1} & \forall t \in [0, t_1) \\ e^* + \beta^{T+1-t}\,(\bar{e} - e^*) & \forall t \in [t_1, T] \\ \bar{e} & \forall t \in (T, \infty)\,. \end{cases} \tag{14}$$

Within $[t_1, T]$, FX intervention satisfies:

$$f_t = \left[\frac{1}{\beta} - \frac{a+c}{a}\right] a e_t + \left[1 - \frac{1}{\beta}\right] a e^* + \bar{z}. \tag{15}$$

which is flat when $\frac{1}{\beta} = \frac{a+c}{a}$, larger and upward-sloping when $\frac{1}{\beta} > \frac{a+c}{a}$, and smaller and downward-sloping when $\frac{1}{\beta} < \frac{a+c}{a}$. T is defined by the feasibility condition:

$$(\bar{e} - e^*)\left\{c\,(T + 1 - t_1) + [a - (a+c)\,\beta]\,\frac{1 - \beta^{T+1-t_1}}{1 - \beta}\right\} = R_0. \tag{16}$$

Corollary 1 (Intervention path) *The intervention period $[t_1, T]$ satisfies:*

$$t_1 \begin{cases} > 0 & \textit{for } R_0 < \overline{R} \\ = 0 & \textit{for } R_0 \geq \overline{R}, \end{cases} \quad \textit{for some } \overline{R} \in \mathbb{R}^{++} \tag{17}$$

$$\lim_{R_0 \to \infty} T = \infty. \tag{18}$$

According to theorem 1, the optimal exchange rate path during $[t_1, T]$ comes directly from the quadratic form of the welfare function and the preference parameter β. By contrast, the

FX intervention path depends on the solution of a separate sub-problem which compares the preference parameter $\frac{1}{\beta}$, capturing the optimal rate of exchange rate depreciation, against $\frac{a+c}{a}$, which captures the rate of depreciation that would be achieved using a constant FX intervention path. In the knife-edge case when $\frac{1}{\beta} = \frac{a+c}{a}$, these two depreciation rates are identical and a constant FX intervention path is optimal.

Figure 3: Full Commitment, Various Values of R

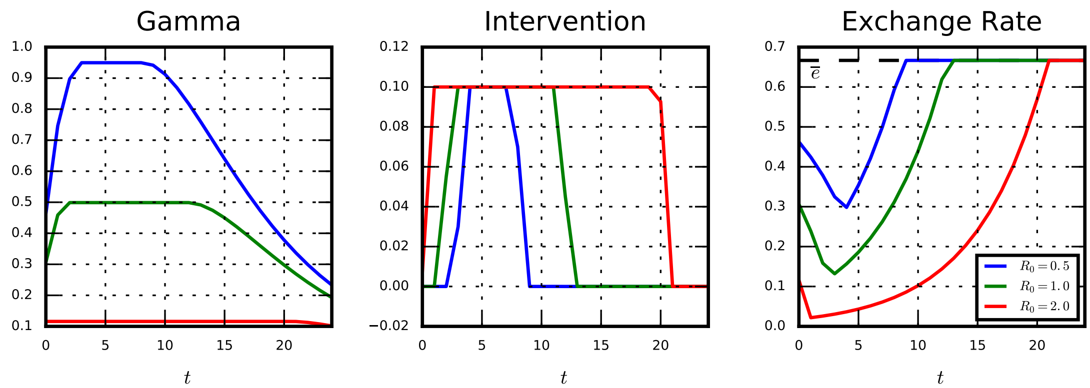


Figure 4: Full Commitment, Various Values of a

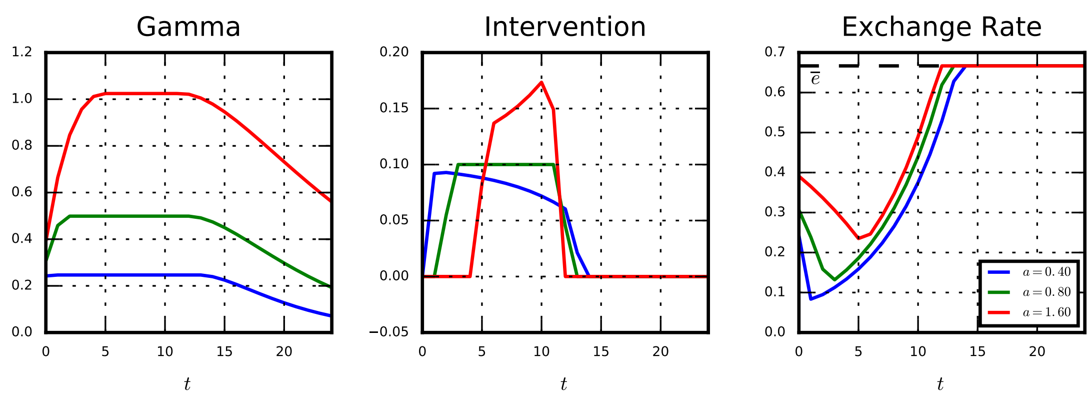
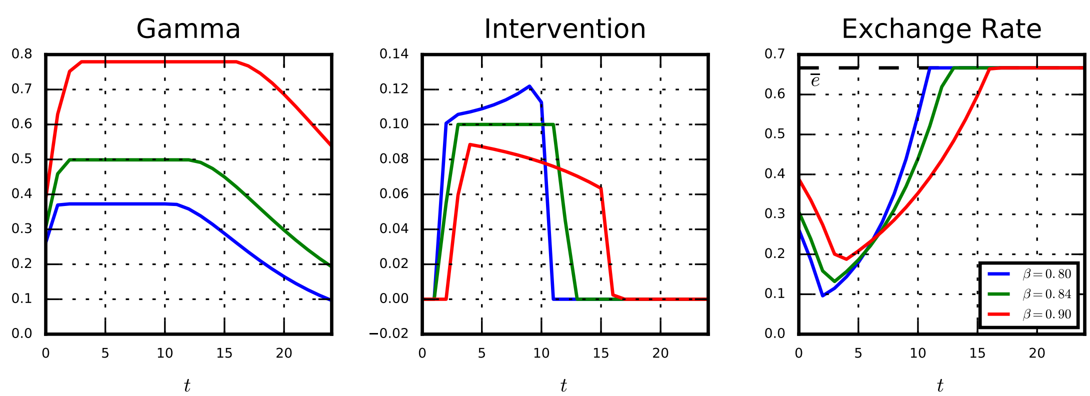


Figure 5: Full Commitment, Various Values of β



The above results are illustrated in figure 3, which again features $\frac{1}{\beta} = \frac{a+c}{a}$, $e^* = 0$, and $\bar{z} = 0.1$. Look first at the graphs for $R_0 = 1$ in order to understand theorem 1. As soon as the shock strikes at date $t = 0$, there is some depreciation of the exchange rate. During the periods $[0, t_1)$, there is no FX intervention but nevertheless, the exchange rate appreciates in the anticipation of future FX intervention. During the periods of intervention $[t_1, T]$, the deviation of the exchange rate from its target e^* grows by a factor of $\frac{1}{\beta}$ in every period. At the end of the intervention period, reserves run out and the exchange rate remains at $\bar{e}$ forever. Because $\frac{1}{\beta} = \frac{a+c}{a}$, FX intervention is completely flat at $f_t = \bar{z}$ during $[t_1, T]$.

Next, to observe corollary 1 in action, see how the graphs in figure 3 vary as the initial level of reserves R_0 varies. The higher is R_0, the earlier that intervention begins and the later that reserves run out, so the longer is the intervention period $[t_1, T]$, and the greater the stabilization of the exchange rate. For initial reserves R_0 sufficiently large, $t_1 = 0$. As $R_0 \to \infty$, $T \to \infty$ and the exchange rate is perfectly stabilized at the target e^*.

Figures 4 and 5 show some comparative statics exercises with respect to a and β. The higher is a, the lower the traction of FX intervention on the exchange rate. Therefore, there is a sharper immediate depreciation above e^* to begin with, and the intervention interval $[t_1, T]$ shrinks, with more intense FX intervention being necessary within that interval. The lower is β, the more the central bank values exchange rate stabilization in early periods relative to later periods, so the central bank intervenes earlier and more aggressively. The time path of FX intervention becomes upward-sloping because the higher optimal depreciation rate $\frac{1}{\beta}$ generates capital outflows which are higher and grow rapidly over time, and FX intervention is increasingly required to offset the impact of these outflows.

Finally, we conclude this section with an observation which motivates the next section.

Remark 1 (Time consistency) *The commitment solution is not time consistent.*

Under full commitment, the central bank promises to intervene in the future because intervention in the future affects exchange rates over a long time period. This logic holds in every period, so if the central bank were allowed to re-optimize tomorrow, it would postpone intervention to the future, and actual intervention would fall below the previously promised level. The easiest way to see this is when the optimal strategy under commitment does not involve any FX intervention during the first few periods, as is the case under several parameter specifications in figures 3, 4 and 5. Then a central bank undertakes no intervention in period $t = 0$ and promises intervention for some intervention interval $[t_1, T]$, where $t_1 > 0$. When the central bank re-optimizes in any future period t, it again undertakes no intervention in period t and promises intervention instead for the interval $[t_1 + t, T + t]$. This continues forever, without any intervention ever actually occurring.

Clearly, investors should never believe the promises of a central bank who can re-optimize its entire future path of intervention in some future period t, ignoring the promises made in previous periods.

4.2. Constant outflows $\bar{z}$ under time consistency

When the central bank has no commitment power at all, its promises regarding FX interventions at future dates are no longer credible. Instead, the solution for FX intervention must be time consistent: the central bank re-optimizes in every period, ignoring the promises of the past. Therefore, neither the optimal policy nor investors' exchange rate expectations depend on such promises. Instead, they depend on the only state variable of the problem: the level of reserves. The FX intervention policy and exchange rate policy can be written as the functions $f(R)$ and $e(R)$ respectively.

While the full-commitment solution establishes an upper bound for the welfare of EME central banks, the time-consistent solution necessarily achieves lower welfare because commitment power is valuable in our model. In practice, every central bank has an intermediate degree of commitment power, lying somewhere between the two extremes.

Definition 5 (Time-consistent problem) *The time-consistent FX intervention policy $f(R)$ and exchange rate policy $e(R)$ satisfy the following conditions:*

- *They are fixed points of the Bellman operator:*

$$v^{TC}(R) = \max_{e,R'} \left\{ -\frac{(e(R) - e^*)^2}{2} + \beta v^{TC}(R') \right\} \tag{19}$$

subject to:

$$\begin{aligned}
\gamma: \quad & e(R) = \frac{1}{a+c}\left[\bar{z} - f(R) + ae(R')\right] \\
\pi: \quad & R - R' = f(R) \\
\psi: \quad & f(R) \geq 0 \\
\phi: \quad & R \geq f(R),
\end{aligned}$$

where the respective multipliers on the (in)equalities are γ, π, ψ, and ϕ, and where the same recursive feasible set for exchange rates that we saw in the full commitment case still applies: $e(R) \geq \bar{e} - \frac{1}{a+c}R$.

- *They are infinitely differentiable: $e(R) \in C^\infty([0,\infty))$ and $f(R) \in C^\infty([0,\infty))$.*

- *Within the set of functions satisfying the above two bullets, select the function that maximizes the central bank's welfare from the perspective of $t = 0$.*

Unlike the full-commitment problem, the time-consistent solution can only be derived as a fixed point to a Bellman operator. In economic terms, the central bank must now take as given the function describing investors' expectations about next period's exchange rate, where those investor expectations come from knowing that the central bank will again face the same Bellman problem at every date in the future[19].

Using this Bellman operator, we can derive the general solution.

Proposition 2 (Generalized Euler equation) *Let $f(R)$ and $e(R)$ be solutions to definition 5 such $f(R) \in (0, R)$ for all R. Then they satisfy the following generalized Euler equation and exchange rate equation:*

$$(e(R) - e^*)[1 + ae_R(R - f(R))] = \beta(e(R - f(R)) - e^*) \tag{20}$$

$$e(R) = \frac{1}{a+c}[\overline{z} - f(R) + ae(R - f(R))]. \tag{21}$$

Theorem 2 (Time-consistent solution) *For $\frac{1}{\beta} > \frac{a+c}{c}$, there exists no time-consistent solution. For $\frac{1}{\beta} \leq \frac{a+c}{c}$, there may exist a time-consistent solution; such a solution satisfies the conditions in proposition 2 and $f(R) \in (0, R)$ for all R. In particular, as $R \to 0$:*

$$e(R) \to \overline{e} \text{ and } e_R(R) \to \frac{\beta - 1}{a}, \tag{22}$$

and as $R \to \infty$:

$$e(R) \to e^* \text{ and } e_R(R) \to 0. \tag{23}$$

Corollary 2 (Reserves undepleted) *For $\frac{1}{\beta} \leq \frac{a+c}{c}$, any time-consistent solution features $R_t > 0$ for all t.*

We first use the above analytical results to provide some intuition regarding the time-consistent solution (if it exists). Then, we turn to our novel numerical solution methodology to characterize the solution.

To build intuition, let us first interpret each term of the generalized Euler equation:

$$\frac{1}{a+c}(e(R) - e^*) = \frac{1}{a+c}\beta(e(R - f(R)) - e^*) - \frac{a}{a+c}e_R(R - f(R))(e(R) - e^*). \tag{24}$$

The left hand side captures the marginal benefit of spending an extra unit of reserves today: the effect of FX intervention on today's exchange rate, $\frac{1}{a+c}$, multiplied by the marginal

[19] Regarding the second bullet, we assume that the functions of interest are infinitely differentiable because we can prove that otherwise, there exists some region of R for which the policy functions $f(R)$ and $e(R)$ are not defined, and therefore that a solution does not exist. Regarding the third bullet, we can prove that for every solution of the Bellman operator where reserves asymptotically tend to 0 as $t \to \infty$, there exists a horizontal translation of the solution which also satisfies the Bellman operator but where reserves asymptotically go to a positive value. The third bullet rules out these horizontal translations as acceptable solutions. The economic interpretation is that we assume that the EME central bank can coordinate investors' expectations to its preferred solution out of the set of time-consistent solutions.

utility of consumption today, $(e(R) - e^*)$. The right hand side has two terms capturing the marginal benefit of leaving an extra unit of reserves for tomorrow. The first term assumes that the extra unit is entirely spent tomorrow, and is the effect of FX intervention on tomorrow's exchange rate, $\frac{1}{a+c}$, multiplied by the discounted marginal utility of consumption tomorrow, $\beta(e(R - f(R)) - e^*)$. The second term is the effect of having an extra unit of reserves tomorrow on today's exchange rate: the strength of the expectations channel, $\frac{a}{a+c}$, multiplied by the change in expectations when the level of reserves left for tomorrow is higher, $e_R(R - f(R))$, multiplied by the marginal utility of consumption today, $(e(R) - e^*)$[20]. The solution will involve $e_R(R - f(R)) < 0$, with higher reserves causing an appreciation in exchange rate expectations.

From this argument, and assuming an interior solution for now (*i.e.*, $f(R) \in (0, R)$ for all R) such that the generalized Euler equation holds, we can see that the capacity for exchange rate stabilization depends on the degree of patience of the central bank. The less patient is the central bank, the more it wants to spend all its reserves today, unless the effect of leaving reserves on exchange rate expectations, $e_R(R - f(R))$, is large in magnitude. Therefore, as β decreases, $e_R(R - f(R))$ must become more negative. Conversely, the more patient is the central bank, the less negative $e_R(R - f(R))$ needs to be. Since expectations must be fulfilled under rational expectations, we can conclude that reserves are more effective in preventing exchange rate depreciations, *i.e.*, $e_R(R)$ is more negative, if the central bank is more impatient[21]. As $R \to 0$, the $e_R(R - f(R))$ necessary for an interior solution tends to the expression $\frac{\beta-1}{a} < 0$.

We next argue that interior solutions cannot exist for very impatient (*i.e.*, low β) central banks. The reason is that there is a limit to how negative $e_R(R - f(R))$ can feasibly be: in particular, from footnote 18 of the previous subsection, we know that the most appreciated exchange rate possible is the one achieved by spending all reserves today, so the $e(R)$ function must lie above that lowest possible exchange rate. As $R \to 0$, this condition reduces to $e_R(0) \geq -\frac{1}{a+c}$. Therefore, for $\frac{\beta-1}{a} < -\frac{1}{a+c} \Leftrightarrow \frac{1}{\beta} > \frac{a+c}{c}$, there exists no interior solution for some nonempty region of R in the neighborhood of 0[22]. What happens instead is that for

[20] It might appear odd that the first term assumes that the extra reserves are entirely spent tomorrow while the second term, as long as $e_R(R - f(R)) > -\frac{1}{a+c}$, assumes that they are not. However, there is no paradox here: the extra reserves are not entirely spent tomorrow, but from the envelope condition (which assumes optimization of the path from tomorrow onward), we can treat them as if they are, for the purpose of calculating the first term.

[21] This result can also be heuristically derived from a related conceptual argument. A central bank with full commitment cares about past promises, today's utility, and the continuation utility; a time-consistent central bank cares only about the last two. Caring about the continuation utility encourages the central bank to preserve reserves and intervene little, breaking past promises, and this happens in all periods so intervention is lower in all periods and exchange rate stabilization is poorer. Therefore, reducing the weight on the continuation utility in the central bank's time-consistent maximization problem can help raise intervention and exchange rate stabilization in all periods.

[22] The assumption of continuous differentiability means that violations that occur in the limit as $R \to 0$ remain violations over R in a neighborhood of 0.

small R, the constraint is binding, $f(R) = R$, and all reserves are used up, while for larger R, they are not.

From this argument, we infer that the time-consistent solution does not exist for very impatient central banks. We can prove that a non-interior solution involves a kink in the $e(R)$ function, which because of the inclusion of the $e_R(R - f(R))$ term in the Euler equation gets transmitted into making the $e(R)$ and $f(R)$ functions undefined for some region of R. The economic interpretation of an undefined region is that for some R, it is impossible to find investor expectations regarding future central bank actions that satisfy subgame-perfection, *i.e.*, the expectations regarding how the central bank would behaves as R varies, are not fulfilled if R does indeed vary off the equilibrium path[23].

Finally, for central banks that are moderately or highly patient, *i.e.*, $\frac{1}{\beta} < \frac{a+c}{c}$, interior solutions do satisfy the feasibility condition $e_R(0) \geq -\frac{1}{a+c}$, and the central bank never uses up its reserves in any period. Therefore, as corollary 2 states, when the time-consistent solution exists, it involves a time path of FX intervention where reserves never get depleted.

Next, we take advantage of the tractable nature of our stylized model in order to solve numerically for the time-consistent solutions $f(R)$ and $e(R)$.

The first step of our numerical procedure is to construct a two-part guess for the shape of the policy functions. We guess that near $R = 0$, the functions behave according to their Taylor series expansion evaluated at $R = 0$[24]. We guess that as $R \to \infty$, the functions $f(R)$ and $e(R)$ converge to the levels implied by the full-commitment solution, with $f(R) \to \overline{z}$, $e(R) \to e^*$ and $e_R(R) \to 0$, so the degree of commitment does not matter for the solution when reserves are large. We also guess that the convergence $e_R(R) \to 0$ is rapid enough that the generalized Euler condition converges to the full-commitment Euler condition for large reserve levels. The second step of our numerical procedure is to put this two-part guess for the shape of the policy functions into a simultaneous equation solver for equations (20) and (21). We use the Levenberg-Marquardt method and cubic splines to interpolate and calculate the derivatives $e_R(R)$.

Figure 6 illustrates our numerical solution for the policy functions $f(R)$ and $e(R)$ in the time-consistent case. The time consistency problem is related to the level of reserves, because that level reflects the proximity of the ZLB constraint on reserves. FX intervention is low near $R = 0$ and converges to $\overline{z}$ as $R \to \infty$. The exchange rate is at the pure float level $\overline{e}$ at $R = 0$, and converges to the exchange rate target $e^* = 0$ as $R \to \infty$.

Figure 7 illustrates the time-consistent solution for the specification $\frac{1}{\beta} = \frac{a+c}{a}$, $e^* = 0$, and $\bar{z} = 0.1$ and setting $R_0 = 1$. FX intervention begins as soon as the outflow episode begins at $t = 0$, and then diminishes over time. Intervention actually occurs in every period, but is

[23]For more details, see the proof of theorem 2 in the appendix.

[24]Appendix subsection 8.2 shows the Taylor expansions for $f(R)$ and $e(R)$ at $R = 0$, as the order of the Taylor expansions are increased. The notion that the functions converge around $R = 0$ is tenable. However, the Taylor expansions for $f(R)$ and $e(R)$ do not converge as $R \to \infty$.

much lower than $\bar{z}$ in all periods. The exchange rate depreciates more in the first period than it does in the full-commitment case. Reserves never run out, because intervention becomes miniscule at low reserve levels.

Figure 6. Time Consistent Policy Functions

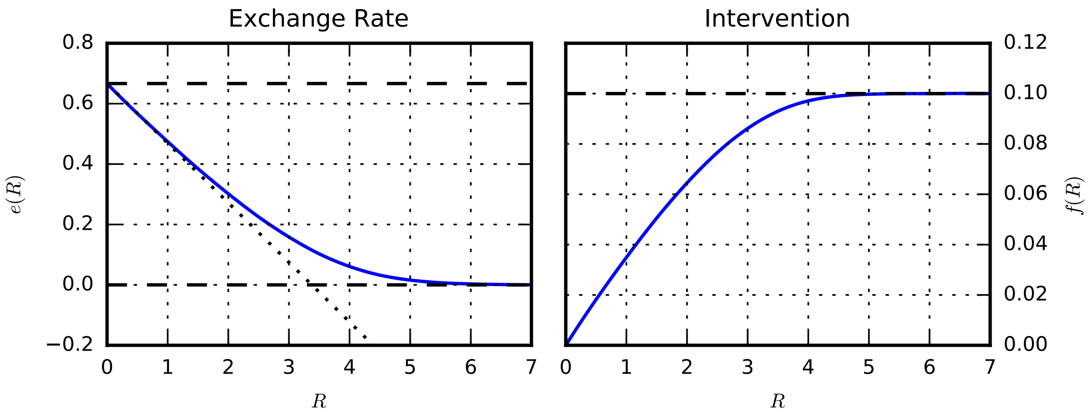
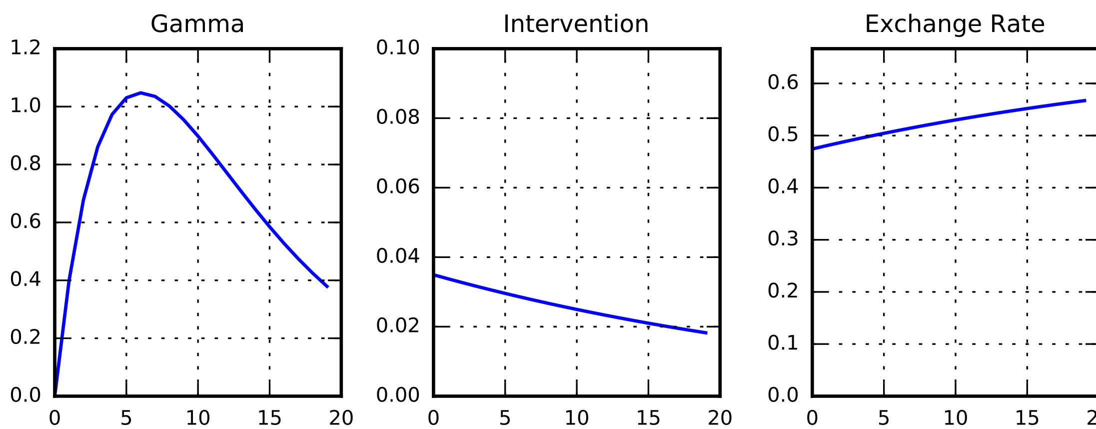


Figure 7: Time Consistent, $R = 1$



The intuition for low FX intervention and poor exchange rate stabilization is that in the time-consistent case, the central bank feels no obligation to fulfil any past promises and it knows that investors' expectations depend positively on the level of reserves left at the end of each period. For both these reasons, the central bank wishes to preserve reserves in each period and retain some room for maneuver in future periods. The result is low FX intervention at all dates. In practice, central banks with low levels of commitment may hesitate to use any of their previously accumulated reserves.

Figure 8 illustrates how the time-consistent solution varies with a. The higher is a, the less traction of FX intervention on the exchange rate today and the stronger is the expectations channel, so the central bank opts to stabilize investors' exchange rate expectations by keeping more reserves in its vaults and by conducting less FX intervention in every period. As $a \to \infty$, FX intervention becomes ineffective, and therefore is never used. Conversely, notice that as

$a \to 0$, the greater the traction of FX intervention on the exchange rate. Therefore, the time-consistent intervention rises towards $\bar{z}$.

Figure 9 illustrates how the time-consistent solution varies with β. The higher is β, the more the central bank values welfare in future periods, so even greater is the tendency for the central bank to keep reserves for the future instead of spending them today. There is a greater immediate depreciation above e^* to begin with. In the limit as $\beta \to 1$, there is no FX intervention in any period at all.

Figure 8: Time Consistent, Various Values of a

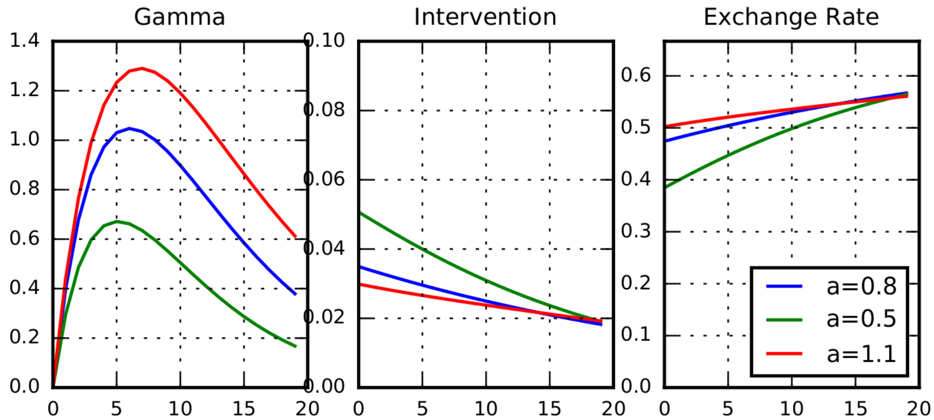
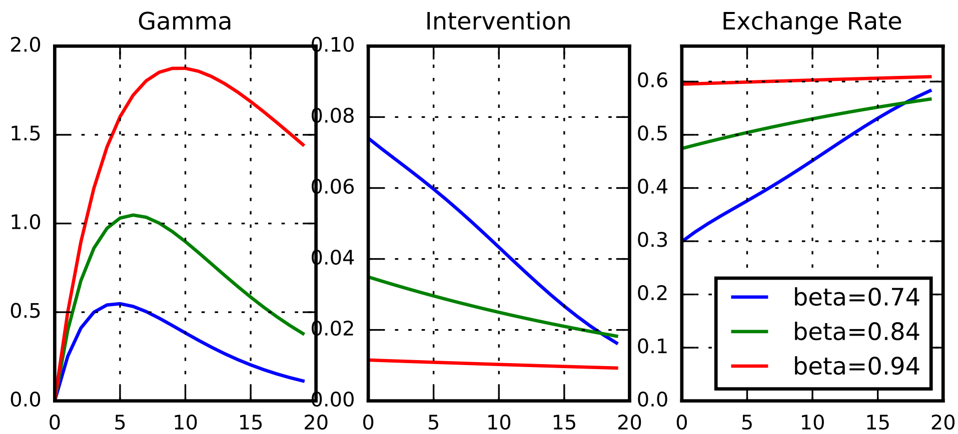


Figure 9: Time Consistent, Various Values of β



Postponement in the use of reserves translates into large immediate exchange rate depreciations when shocks strike. Therefore, relative to the full-commitment case, central banks experience a reduction in welfare when they lack the power to commit to future FX intervention policies, and the reduction in welfare may be severe for some parameter specifications. In this light, we turn next to possible remedies.

4.3. Constant outflows $\overline{z}$ with simple FX intervention rules

In the preceding section, we have seen that lack of commitment prevents an EM central bank from intervening to sustain the exchange rate as aggressively as it would do if it were able to commit. In this section we consider a central bank with a partial degree of commitment power. In other words, it does have the ability to commit to some simple FX intervention rules which are easy to communicate to investors, but it remains unable to commit to the general full-commitment path. We wish to assess whether, by committing to simple rules, a central bank can raise its welfare above the purely discretionary time-consistent level.

We consider two rules: an exchange rate peg and a volume intervention rule, both indexed by a parameter $\kappa \in [0, 1]$ which captures how aggressive intervention is under the rule (the higher is κ, the more aggressive is the intervention). We assume that $e^* = 0$.

Definition 6 (Exchange rate peg) *An exchange rate peg is characterized by:*

$$e_t = (1 - \kappa)\,\overline{e} \text{ until } R_t = 0. \tag{25}$$

Proposition 3 (Exchange rate peg solution) *There exists a period T in which the peg breaks because reserves have run out. The exchange rate jumps to $\overline{e}$ from T onwards. FX intervention and the breaking time T jointly satisfy:*

$$f_t = \begin{cases} \bar{z} & \text{for } t < T - 1 \\ \bar{z} + \overline{e}\,[a - (a + c)(1 - \kappa)] & \text{for } t = T - 1 \\ 0 & \text{for } t \geq T \end{cases} \tag{26}$$

$$\bar{z}\,(T - 1) < R_0 - \overline{e}\,[a - (a + c)(1 - \kappa)] \leq \bar{z}T. \tag{27}$$

Definition 7 (Volume intervention) *A volume intervention rule is characterized by:*

$$f_t = \kappa\bar{z} \text{ until } R_t = 0. \tag{28}$$

Proposition 4 (Volume intervention solution) *There exists a period T in which reserves run out. The exchange rate, FX intervention and the breaking time T jointly satisfy:*

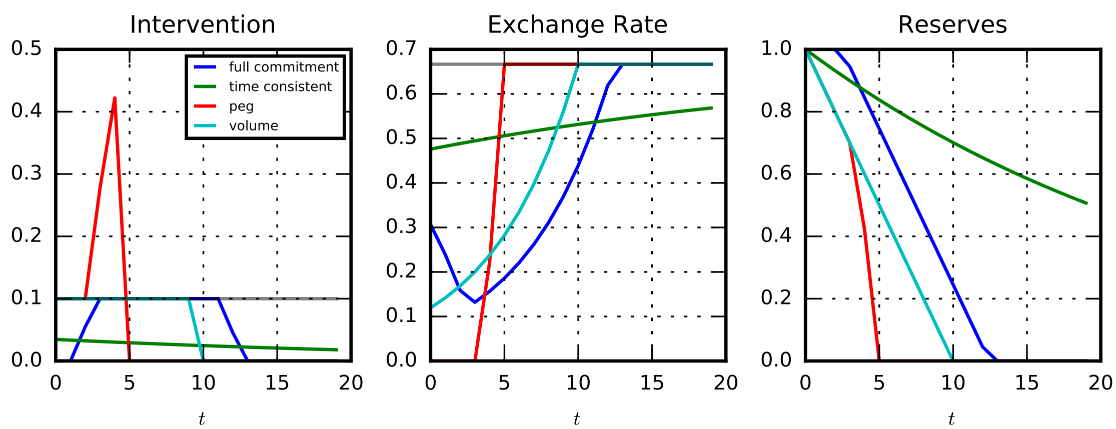
$$e_t = \begin{cases} \left(\frac{a}{a+c}\right)^{T+1-t}\overline{e} & \text{for } t < T \\ \overline{e} & \text{for } t \geq T \end{cases} \tag{29}$$

$$f_t = \begin{cases} \min\{\bar{z}, R_t\} & \text{for } t < T \\ 0 & \text{for } t \geq T \end{cases} \tag{30}$$

$$\bar{z}\,(T - 1) < R_0 \leq \bar{z}T. \tag{31}$$

Figure 10 illustrates the time paths for FX intervention $\{f_t\}_{t=0}^{\infty}$ and exchange rates $\{e_t\}_{t=0}^{\infty}$ for the simple FX intervention rules as well as for the full-commitment and time-consistent solutions, for the specification $\frac{1}{\beta} = \frac{a+c}{a}$, $e^* = 0$, $\bar{z} = 0.1$, and $\kappa = 1$. The exchange rate peg keeps the exchange rate at the target level e^* for some time by fully offsetting the capital outflow shock $\bar{z}$, but intervention spikes in the final period $T-1$, when investors pull their money out of the country in anticipation of the break of the peg. This spike in intervention at the end of the peg curtails the duration of the peg. The volume intervention rule also fully offsets the capital outflow shock $\bar{z}$, but the exchange rate now depreciates smoothly at rate $\frac{a+c}{a}$. In the figure, this rate is the same as $\frac{1}{\beta}$, but notice that even in this knife-edge case, the volume intervention rule involves intervention starting and ending at suboptimal times relative to the full-commitment case. Notice that both of the simple FX intervention rules generate higher intervention than the time-consistent case at the beginning of the outflow episode. Therefore, the immediate depreciation in period $t = 0$ is lower with the simple rules than it is in the time-consistent case.

Figure 10. Policy Comparisons for the Deterministic Case

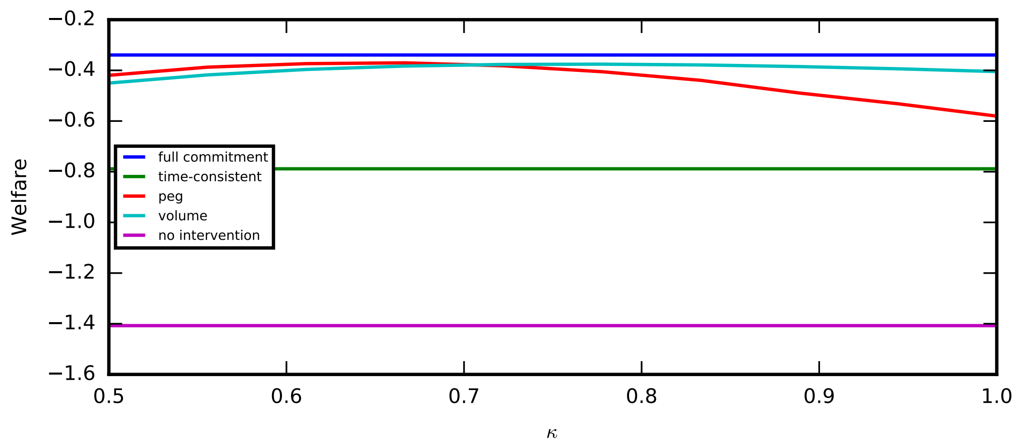


Proposition 5 (Welfare comparison) *The central bank's welfare depends on its commitment power and communication strategy:*

- *The full-commitment solution generates a higher welfare than any other solution.*
- *The time-consistent solution generates a higher welfare than zero intervention, but represents a lower bound for welfare within the set of optimal solutions under varying degrees of commitment power.*
- *There exists* $\kappa \in [0, 1]$ *such that the exchange rate peg and/or volume intervention rule generate higher welfare than the time-consistent solution.*

Figure 11 illustrates the welfare levels under full, zero, and partial commitment, for various levels of κ. The exchange rate peg and volume intervention rule achieve higher welfare than the time-consistent solution, primarily because they prevent the large immediate depreciation associated with time consistency. Therefore, if a central bank has the partial commitment power needed to commit to simple FX intervention rules, and it has less of a history in undertaking heavy interventions and/or is facing investors with less anchored expectations, then it should commit to such rules instead of using pure discretion.

Figure 11. Welfare Comparisons for the Deterministic Case



5. The Persistence of the Shock

In this section, we show that the optimal policy responses by EME central banks depend on their assessment regarding the persistence of the shock. For shocks that are assessed to be temporary, or whose immediate severity is expected to dissipate over time (such as the outflow episodes in many EMEs at the start of the global financial crisis described in section 2), the optimal FX intervention policy does not depend much on the degree of commitment of the central bank, and involves fully offsetting the outflow shock and stabilizing the exchange rate as much as possible. For shocks that are assessed to be more persistent, even if the outflow in each period may be small (such as the "taper tantrum" episodes in section 2, when some investors and central banks feared that additional and protracted U.S. monetary policy tightening was imminent), the likelihood of the ZLB binding at some future date is higher if the central bank fully offsets the shock indefinitely, and investors' expectations about future interventions at low reserve levels feed into exchange rate expectations and optimal policy today. Therefore, the absence of credible promises to intervene in the future becomes more costly in welfare terms, and the time consistency problem is larger. Under full commitment, FX intervention is optimally postponed into the future, but the time-consistent solution diverges from this benchmark, producing low FX intervention and poor exchange rate stabilization instead. In this context, simple FX intervention rules (as in the Brazilian outflow episode in section 2) may achieve better stabilization than pure discretion.

Throughout this section, we focus on the stochastic outflow shock described in definition 2, where $z_t(s^t) = \overline{z} > 0$ for $t = 0$, and then in every period, the shock has a probability p of persisting at the same level into the next period, and a probability $1 - p$ of falling to zero and remaining there forever. The pure float exchange rate is now $\overline{e}(s^t) = \frac{z_t(s^t)}{a(1-p)+c}$, which is positive in periods when the shock strikes and zero forever once the shock disappears.

This specification of the model is more intuitive than the constant outflows case, because the exchange rate target $e^* = 0$ and the long-run exchange rate level coincide. While our chosen stochastic process clearly does not cover the diverse array of outflow shocks observed in practice, it does enable us to begin exploring the relationship between shock persistence and time consistency, while also preserving the tractability of the model (which in turn helps achieve numerical solutions in the time-consistent case).

5.1. Analytics under full commitment and time consistency

We first summarize how the central bank optimization problems are amended for the new outflow shock, before moving on in later subsections to a comparison of the optimal policy paths for various degrees of shock persistence p. We draw heavily here on the concepts in subsections 4.1 and 4.2, and the notation in section 3.

Under full commitment, the central bank now credibly promises the entire history-contingent path of FX interventions and exchange rates $\{f_t(s^t), e_t(s^t)\}_{t=0}^{\infty}$. Notice that the sequence problem can again be written in the Bellman form, because we can write the exchange rate $e_t(s^t)$ in each period t after history s^t as a function of three variables: the shock $z_t(s^t)$ in the same period, the FX intervention $f_t(s^t)$ in the same period, and the expectation of next period's exchange rate $E_{s^t} e_{t+1}(s^{t+1})$. The latter variable is now a sufficient statistic capturing the effect of the entire promised history-contingent path of future FX interventions $\{f_{t+i}(s^{t+i})\}_{i=1}^{\infty}$ on $e_t(s^t)$.

Definition 8 (Commitment problem) *The commitment solution comprises paths of FX intervention $\{f_t(s^t)\}_{t=0}^{\infty}$, exchange rates $\{e_t(s^t)\}_{t=0}^{\infty}$ and reserves stocks $\{R_{t+1}(s^t)\}_{t=0}^{\infty}$ which solve:*

$$W^{FC}(R_0, s_0) = \max_{\{e_t(s^t), f_t(s^t)\}_{t=0}^{\infty}} - E_{s_0} \sum_{t=0}^{\infty} \beta^t \frac{(e_t(s^t) - e^{\star})^2}{2} \tag{32}$$

subject to, for each period t:

$$\begin{aligned}
\Gamma_t(s^t) : \quad & \quad e_t(s^t) = \frac{1}{a+c}\left[z_t(s^t) - f_t(s^t) + aE_{s^t} e_{t+1}(s^{t+1})\right] \\
\Pi_t(s^t) : \quad & R_t(s^{t-1}) - R_{t+1}(s^t) = f_t(s^t) \\
\Psi_t(s^t) : \quad & \quad f_t(s^t) \geq 0
\end{aligned}$$

$$
\begin{aligned}
\Phi_t(s^t): &\quad R_t(s^{t-1}) \geq f_t(s^t) \\
\Lambda: &\quad R_0 \geq \sum\nolimits_{t=0}^{\infty} f_t(s^t),
\end{aligned}
$$

where the respective multipliers on the (in)equalities are $\Gamma_t(s^t)$, $\Pi_t(s^t)$, $\Psi_t(s^t)$, $\Phi_t(s^t)$, and Λ. The initial level of reserves R_0 and the initial state of nature s_0 are given exogenously.

Lemma 5 (Bellman representation) *The commitment problem can be written in the following Bellman form:*

$$
v^{FC}(R, \mu, s_{-1}) = \max_{e(s), f(s), \mu'(s)} E_{s_{-1}} \left\{ -\frac{(e(s) - e^*)^2}{2} + \beta v^{FC}(R - f(s), \mu'(s), s) \right\} \tag{33}
$$

subject to:

$$
\begin{aligned}
\gamma(s): &\quad e(s) = \frac{1}{a+c}[z(s) - f(s) + a\mu'(s)] \\
\pi(s): &\quad R - R'(s) = f(s) \\
\psi(s): &\quad f(s) \geq 0 \\
\phi(s): &\quad R \geq f(s) \\
\delta(s): &\quad \mu = E_{s_{-1}} e(s) \\
\sigma(s): &\quad \mu'(s) \geq \bar{e} - \frac{1}{a+c} R'(s),
\end{aligned}
$$

where s represents the state of nature today, s_{-1} represents the state of nature in the previous period, and the respective multipliers on the (in)equalities are denoted by $\gamma(s)$, $\pi(s)$, $\psi(s)$, $\phi(s)$, δ, and $\sigma(s)$, and where the final constraint is derived recursively from the definition of the feasible set $M(R, s_{-1})$:

$$
M(R, s_{-1}) = \left\{ \begin{array}{c} \mu(s) : \mu(s) = \frac{1}{a+c}(z(s) - f(s) + a\mu'(s)) \\ \text{for some } \mu'(s) \in M(R', s),\ f(s) = R - R'(s) \in [0, R],\ \text{and } s \succ s_{-1} \end{array} \right\}.
$$

Proposition 6 (Euler equation) *At any date t such that $f_t(s^t)$ is in the interior of its feasible set:*

$$
\Gamma_t\left(s^t\right) = E_{s^t} \Gamma\left(s^{t+1}\right) \tag{34}
$$

$$
\frac{c}{a+c}\left(e_t\left(s^t\right) - e^*\right) + \frac{a}{a+c} E_{s^{t-1}}\left(e_t\left(s^t\right) - e^*\right) = \beta E_{s^t}\left(e_{t+1}\left(s^{t+1}\right) - e^*\right). \tag{35}
$$

The equations hold with the inequality "$\geq$" when reserves have become depleted, $R_{t+1}(s^t) = 0$.

The new Euler equation for the full-commitment case appears unusual, and is in fact related to the desire by the central bank to use future promises of FX intervention in an attempt to offset the non-contingent nature of reserves. Given the nature of the shock, reserves are more valuable in future histories where the shock persists than in future histories where the shock has disappeared. In order to "redistribute" reserves across these future histories, the central bank achieves any given level of exchange rate stabilization today by promising higher interventions at future dates when reserves become less valuable (after the shock has disappeared) and lower interventions at future dates when reserves become more valuable (after the shock persists).

As the above equation shows, this kind of commitment power is valuable and stabilizes the exchange rate path well after shocks. Suppose that a severe shock strikes the economy in period t and history s^t. The term on the right hand side, representing the expected future deviation of the exchange rate $e_{t+1}(s^{t+1})$ from the target e^*, is determined by a weighted sum of the large exchange rate deviation after the severe shock and the smaller exchange rate deviation that could have occurred in period t had other less severe shocks struck. Therefore, the future expected exchange rate deviations remain "anchored," in the sense of being partially cushioned from adverse shock realizations today[25].

While such "anchoring" of exchange rate paths may be the ultimate goal of a well-communicated managed float regime, it may not be a realistic benchmark for most central banks, as it requires heavy intervention after the outflow shock has actually disappeared forever[26]. Therefore, in the next subsection, we compare the full-commitment and time-consistent solutions assuming that intervention is not allowed in the full-commitment solution after the shock has ended. In a later subsection, for completeness in our exposition, we separately characterize the full-commitment solution without this restriction.

Under time consistency, the central bank's FX intervention and exchange rate policy functions now follow the formulations $f(R, z)$ and $e(R, z)$ respectively, where z represents today's value of the outflow shock. In this case, no past promises by the central bank need to ever be fulfilled, so once the outflow shock has ended, it is trivial to show that there is no FX intervention, $f(R, 0) = 0$, and the exchange rate remains forever at the target, $e(R, 0) = e^*$. Therefore, we only need to find the functions $f(R, \bar{z})$ and $e(R, \bar{z})$ that represent the optimal policy while the outflow shock $\bar{z}$ is continuing to strike. It turns out that these functions are the fixed points of a Bellman operator which is a slight modification of the Bellman operator that has already been solved for the deterministic case in subsection 4.2.

Definition 9 (Time-consistent problem) *The time-consistent FX intervention policy $f(R, \bar{z})$ and exchange rate policy $e(R, \bar{z})$ satisfy the following conditions:*

[25] This result echoes the result in the inflation targeting literature that credible central banks can ensure that one-off inflation shocks, despite affecting the inflation rate today, do not feed into inflation expectations. Bernanke (2007) explains the issue clearly: "With inflation expectations well anchored, a one-time increase in energy prices should not lead to a permanent increase in inflation but only to a change in relative prices." Gürkaynak, Levin, and Swanson (2010) argue that inflation targeting makes long-run inflation expectations less sensitive to short-term economic news.

[26] Consistent with other papers in the optimal policy literature, the demands on policymaker credibility are higher (and usually, even more unrealistic) in a stochastic environment than in the deterministic case.

- *They are fixed points of the Bellman operator:*

$$v^{TC}(R,\bar{z}) = \max_{e(R,\bar{z}),R'(R,\bar{z})} \left\{ -\frac{(e(R,\bar{z})-e^*)^2}{2} + \beta p v^{TC}(R'(R,\bar{z}),\bar{z}) \right\} \tag{36}$$

  subject to:

$$\begin{aligned}
\gamma: &\quad e(R,\bar{z}) = \frac{1}{a+c}\left[\bar{z} - f(R,\bar{z}) + ape(R'(R,\bar{z}),\bar{z}))\right] \\
\pi: &\quad R - R'(R,\bar{z}) = f(R,\bar{z}) \\
\psi: &\quad f(R,\bar{z}) \geq 0 \\
\phi: &\quad R \geq f(R,\bar{z}),
\end{aligned}$$

  where the respective multipliers on the (in)equalities are γ, π, ψ, and ϕ, and where the same recursive feasible set for exchange rates that we saw in the full commitment case still applies: $e(R) \geq \overline{e} - \frac{1}{a+c}R$.

- *They are infinitely differentiable: $e(R,\bar{z}) \in C^\infty([0,\infty))$ and $f(R,\bar{z}) \in C^\infty([0,\infty))$.*

- *Within the set of functions satisfying the above two bullets, select the function that maximizes the central bank's welfare from the perspective of $t=0$.*

Proposition 7 (Generalized Euler equation) *Let $f(R,\bar{z})$ and $e(R,\bar{z})$ be solutions to definition 9 such $f(R,\bar{z}) \in (0,R)$ for all R. Then they satisfy the following generalized Euler equation and exchange rate equation:*

$$(e(R,\bar{z})-e^*)\left[1+ape_R(R-f(R,\bar{z}),\bar{z})\right] = \beta\left(pe(R-f(R,\bar{z}),\bar{z})-e^*\right) \tag{37}$$

$$e(R,\bar{z}) = \frac{1}{a+c}\left[\bar{z}-f(R,\bar{z})+ape(R-f(R,\bar{z}),\bar{z})\right]. \tag{38}$$

Corollary 3 (Reserves undepleted) *Suppose that $e^*=0$. For $\frac{1}{\beta} \leq \frac{(a+c)p}{a(1-p)+c}$, any time-consistent solution features $R_t > 0$ for all t.*

This solution produces new comparative statics of the time-consistent solution with respect to the shock persistence parameter p. To build intuition, observe first that $p<1$ qualitatively changes the problem in two ways. First, it weakens the expectations channel of exchange rate stabilization (see the a terms multiplied by p). This change both reduces the effect of the shock $\bar{z}$, because it is not expected to last forever anymore, and the effect of expected future intervention when the shock strikes, because there is a lower probability that the intervention materializes. Second, the central bank discounts more heavily future histories where the shock persists (see the β term multiplied by p). This change makes the central bank more impatient, more willing to spend reserves today instead of in the future, and therefore (drawing on the arguments in subsection 4.2) better able to stabilize today's exchange rate.

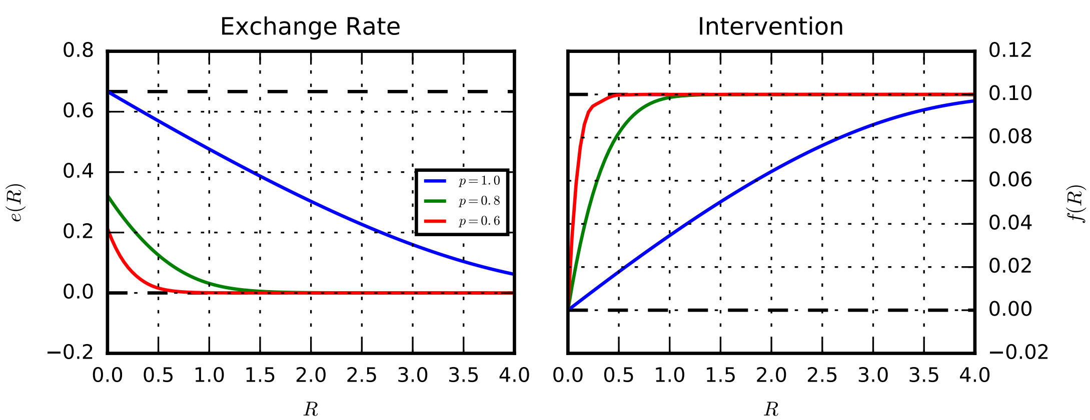
For a lower shock persistence p, the pure-float exchange rate $\overline{e}\left(s^t\right)$ is more appreciated. In addition, as $R \to 0$, we can derive that $e_R\left(R - f\left(R, \bar{z}\right), \bar{z}\right)$ tends to the expression $\frac{\beta p-1}{ap} < 0$, which is more negative for lower p. Therefore, in the time-consistent case, FX intervention is higher, and exchange rate stabilization superior, after a more temporary shock.

5.2. Shock persistence p and the time consistency problem

Proposition 8 (Shock persistence) *When $p = 1$, the full-commitment and time-consistent solutions are divergent, as described in section 4. As $p \to 0$, the two solutions coincide.*

Figure 12 illustrates the optimal policy functions $f\left(R, \bar{z}\right)$ and $e\left(R, \bar{z}\right)$ for different values of p. For each level of p, the intervention rule converges to the full commitment intervention $\overline{z}$ when the level of reserves R is large enough. As the persistence p declines, the time-consistent and full-commitment solutions appear to converge for lower levels of reserves. In the limit as $p \to 0$ and the shock becomes a pure one-off, the functions $f\left(R, \bar{z}\right)$ and $e\left(R, \bar{z}\right)$ become vertical at $R = 0$, which proves that the solutions under full commitment and time consistency become identical for all levels of reserves.

Figure 12. Time Consistent Policy Functions, Various Values of p



Figures 13 and 14 illustrate the optimal FX intervention and exchange rate time paths for full commitment, time consistency, and simple intervention rules, for the persistence levels $p = 0.8$ and $p = 0.6$ respectively. These graphs can be compared to figure 10, which captures the persistence level $p = 1$. As described in the previous subsection, we assume for expositional purposes that there is zero FX intervention after the shock has ended (which is the optimal result in the time-consistent case, and a realistic constraint on the full-commitment case). As the degree of persistence p declines, the full-commitment intervention begins earlier, overlapping with the volume rule, and the time-consistent solutions for both intervention and the exchange rate become closer to the full-commitment solutions.

Figure 13. Policy Comparisons for Shock Persistence $p = 0.8$

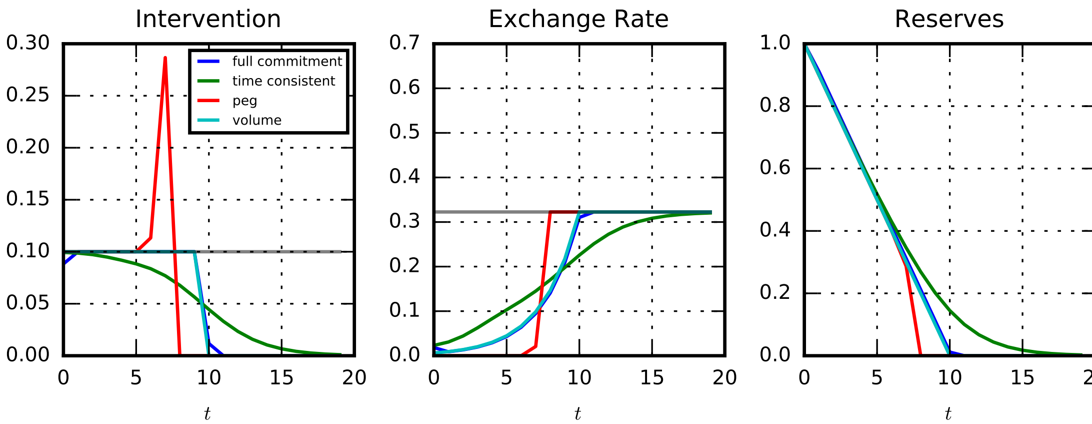
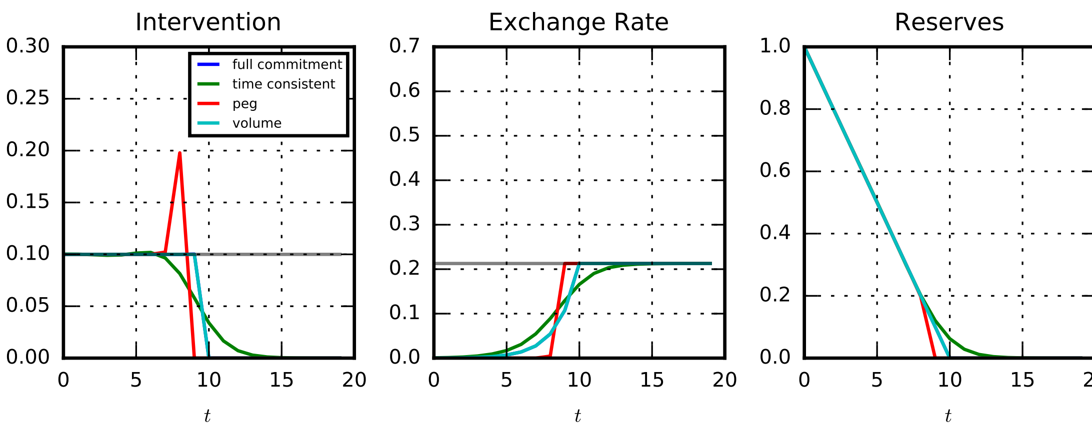


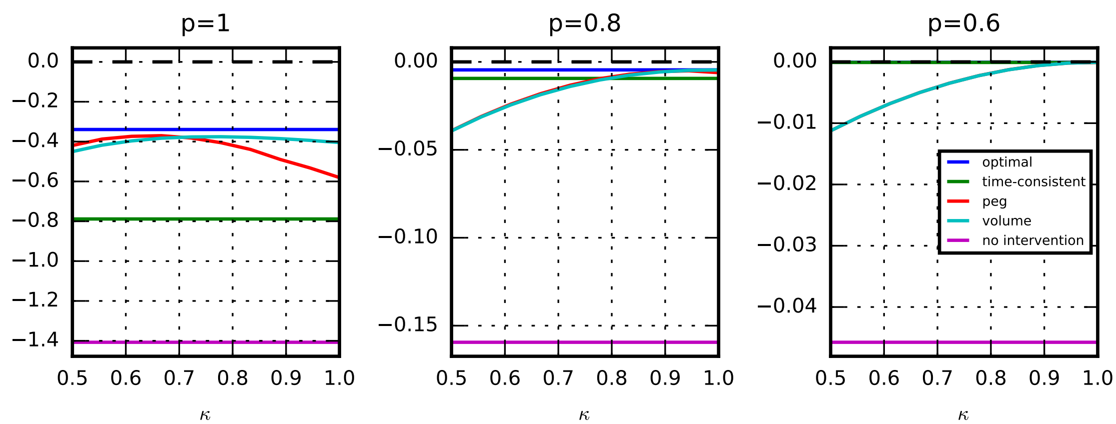
Figure 14. Policy Comparisons for Shock Persistence $p = 0.6$



The time consistency problem is more severe for more persistent shocks. The intuition for this result is as follows. When the shock is a pure one-off ($p = 0$), the shock is expected to go to zero for all future periods, so no intervention is expected in the future, and investors' expectations about next period's exchange rate $E_{s^t} e_{t+1}(s^{t+1})$ are unaffected by the central bank's commitment power. Therefore, the central bank solves a simple one-period problem today, which yields an identical policy in both the full-commitment and time-consistent cases. When the shock is very persistent (p near 1), it is expected to continue for many periods and has a higher net present value, so investors' expectations regarding the central bank's future interventions become important. The central bank wishes to preserve reserves to last throughout a long outflow episode. The central bank with full commitment achieves this by credibly promising to begin an aggressive intervention strategy later even if reserves thereby run out. But the time-consistent central bank cannot do this in the presence of the ZLB on reserves, because it will choose to preserve reserves rather than spend them when reserves are about to run out. Correspondingly, investors are skeptical of regarding any promises to intervene aggressively in the future. Therefore, in a model with a ZLB on reserves, the larger is the persistence parameter p, the more damaging is a lack of commitment power[27].

In subsection 4.3, we argued that simple intervention rules can improve welfare above the purely discretionary time-consistent case. Now, we can qualify that claim: rules are likely to achieve welfare gains above discretion when shocks are very persistent, because for such shocks, commitment power is very valuable. Figure 15 illustrates the welfare levels under full commitment, time consistency, and simple intervention rules across various levels of κ, for the cases when $p = 1$, $p = 0.8$ and $p = 0.6$. When the degree of persistence p declines, welfare under time consistency gets closer to the full-commitment welfare, and there is a reduction in the range of values of κ for which the simple FX intervention rules yield higher welfare than the purely discretionary time-consistent solution. Therefore, for more temporary shocks, the scope for welfare improvements through the use of rules diminishes.

Figure 15. Welfare Comparisons for Various Values of Shock Persistence p



5.3. Post-shock intervention under full commitment

As promised, for completeness, we next characterize the full-commitment solution without the restriction that intervention must stop when the shock does. This section may appear esoteric at first glance, but actually yields two lessons. First, the unrestricted full-commitment policy does involve one feature that echoes the finding of Eggertsson and Woodford (2003): the central bank would like to promise FX intervention after the shock ends in order to stabilize exchange rates while the shock is underway[28]. Second, the path thus derived is

[27]This argument generally applies to other models with different welfare functions and exchange rate equations, provided that there is a ZLB on reserves and provided that the probability of reserves becoming fully depleted in the future is above zero if the central bank were to apply the full-commitment policy derived by ignoring the ZLB.

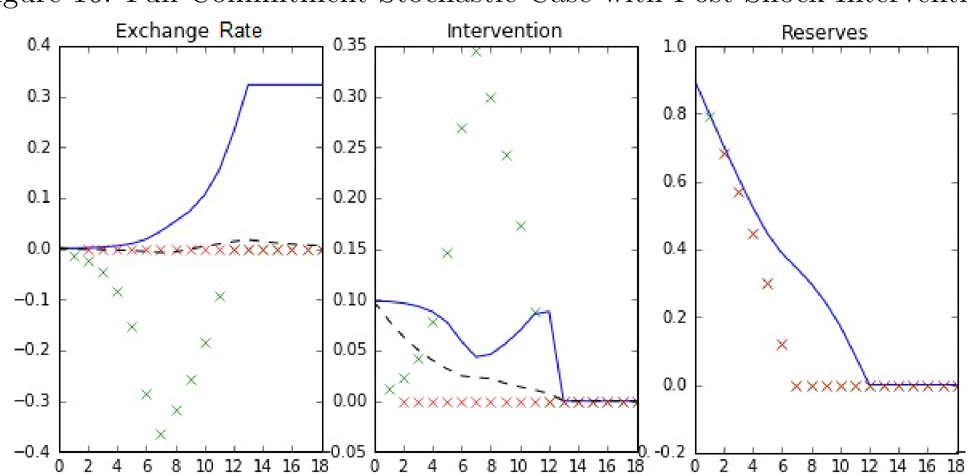
[28]Eggertsson and Woodford (2003) show that for a central bank facing a ZLB constraint on interest rates, welfare can be brought to the constrained-optimal level through commitment to a price-level targeting rule

rather unrealistic when applied to the FX intervention problem, and would require significant commitment power to implement. Therefore, the previous subsection's benchmark, that intervention is not allowed after the shock has ended, is the more natural one.

Lemma 6 (Post-shock intervention) *The full-commitment solution in the stochastic case involves FX intervention in the period after the shock ends, and not at all thereafter.*

Figure 16 illustrates this result. The blue lines plot FX intervention and the exchange rate in period t conditional on the shock $z_t(s^t) = \overline{z}$ in period t, while the green dots plot FX intervention and the exchange rate in period t conditional on the shock stopping in period t, *i.e.*, $z_{t-1}(s^{t-1}) = \overline{z}$ and $z_t(s^t) = 0$. The central bank supports the exchange rate path during the outflow episode by committing to intervene heavily in the period after the shock ends (when reserves are no longer valuable for any future use, but when a promised exchange rate appreciation can help support expectations of investors in the previous period, while the shock was still occurring)[29]. From period $t = 0$ to $t = 7$, an increasing amount of intervention is promised in the period right after the shock ends. However, after period $t = 7$, so many reserves have been used up during the outflow episode already that the post-shock intervention cannot be as high as the central bank would like to commit to. Therefore, there is a kink in the FX intervention and exchange rate functions. The expectation of this kink in the function at $t = 7$ then generates a feedback effect onto investors' expectations in prior periods, which the central bank must take account of. It turns out that the entire FX intervention path is no longer flat at $\overline{z}$ as it used to be in the deterministic case, and instead exhibits a counter-intuitive (and unrealistic) dip and rise during the intervention period.

Figure 16. Full Commitment Stochastic Case with Post-Shock Intervention



after the ZLB is no longer binding. Notice that the parallel is not exact, however: our model has a ZLB on reserves, which is a stock variable, not on a flow variable such as the interest rate.

[29]The central bank will never optimally commit to intervene more than one period after the shock ends: once the shock ends, there is no need to stabilize the exchange rate any further, so no promises of future FX interventions are needed.

6. The Composition of the FX Market

In this section, we show how optimal policy responses depend on the composition of investors in the FX market. In our baseline model, the only participants in the FX market were speculators facing limits to arbitrage: beyond the exogenous shock $z_t(s^t)$, these speculators moved funds between domestic and foreign currencies depending on the expected depreciation $(E_{s^t}e_{t+1}(s^{t+1}) - e_t(s^t))$ that they calculated using rational expectations, but they moved funds in finite quantities so that the uncovered interest parity condition did not hold. As a result, FX intervention always had traction on the exchange rate, and in designing the optimal policy, the central bank understood that the time path of expected depreciations affected the endogenous component of capital outflows in every period, and thereby the FX interventions necessary in every period to stabilize the exchange rate at any desired level.

However, narrative histories of outflow episodes highlight that EME central banks are often concerned about other categories of investors in the FX market. Conceptually, we can rationalize this notion by observing that groups of investors with informational or data-processing limitations, or with different objective functions, may follow behavioral rules which diverge from the limited-arbitrage rational-expectations-based capital outflows of our baseline model. The presence of such groups of investors alters the behavior of both the speculators and the central bank who already existed in the baseline model. In what follows, we focus on two kinds of heuristic rules that FX market participants may follow, and we draw out their implications for the central bank's optimal policy responses. For purposes of clarity, throughout this section we focus on the deterministic outflow shock with constant outflows $\bar{z}$, but the results carry over to the stochastic outflows case.

Firstly, we consider the possibility of "panickers" in the FX market: foreign investors who sell the domestic currency or withdraw their FX lending to domestic banks when they observe the central bank's reserves being drawn down. If the propensity to panic is severe, large planned FX intervention can become "counterproductive" at the margin and the exchange rate can become destabilized as a result (such a behavior may motivate some central banks to keep their FX interventions non-public in outflow episodes). Secondly, we analyze "Knightian flight," when a group of unsophisticated investors based in a foreign country abhor exchange rate risk, and hold the domestic currency purely because they (incorrectly) think that the exchange rate will forever be pegged to their own currency; subsequently, they sell all their holdings as soon as the peg breaks. If the domestic banking system is dependent on such investors, their exit can cause a banking crisis (such a concern may have been one worry among several during the global financial crisis). The existence of such investors is a nuisance in the full-commitment case, but provides commitment power to prevent depreciations in the time-consistent case, and under some parameter configurations may improve welfare.

6.1. Panickers

Consider a new group of foreign investors who focus solely on the depletion of central bank reserves, and withdraw their own lending to the domestic economy when reserves decline. This behavior may be motivated in two ways. Firstly, speculators without access to the full set of information available in our baseline model setup, or with a limited ability to process this information, may instead focus solely on the depletion of central bank reserves as a proxy for future exchange rate stabilization, based upon the notion that lower reserves at the end of any period mechanically reduces the capacity for future FX interventions[30]. Secondly, foreign investors who provide FX lending to the domestic banking system may believe that their loans are (explicitly or implicitly) backed by the central bank's FX reserves, so that a depletion of those reserves makes it riskier to lend to domestic banks.

We model the capital outflows k_t^P by these "panickers" as follows:

$$k_t^P = \alpha \left(R_t - R_{t+1}\right) + \frac{\left(R_t - R_{t+1}\right)^2}{2\theta} = \alpha f_t + \frac{f_t^2}{2\theta}, \tag{39}$$

which is increasing in the reserves depletion, or in other words the FX intervention, observed in period t. The linear and quadratic terms reflect different forms of panic: the linear term represents a constant outflow per unit of reserves depletion, while the quadratic term represents accelerating panic for larger reserve depletions (perhaps because panickers take large depletions as a proxy for especially worrying information). Higher α and lower θ correspond to a higher propensity to panic. In this subsection, for purposes of clarity, we will consider separately variations in α and θ.

Using equations (1), (2), (3) and (39) in the amended balance of payments identity:

$$k_t + k_t^P \equiv ca_t + f_t, \tag{40}$$

we can derive the amended set of equations characterizing the feasible set of the model.

Definition 10 (Model with panickers) *The reduced-form version of the model is described by the equations for the exchange rate and FX intervention:*

$$e_t = \frac{1}{a+c}\left(\bar{z} - h_t + a e_{t+1}\right) \tag{41}$$

[30]As shown in sections 4 and 5, this kind of behavior is not optimal under full-information rational expectations for investors maximizing their returns from exchange rate speculation, because such investors should ideally based their capital outflow decisions not just on the level of reserves but also on promised future FX interventions (in the full-commitment case), an understanding of the central bank's optimization problem in future periods (in the time-consistent case), and/or the existence of FX intervention rules (in the partial commitment case).

$$h_t = (1-\alpha) f_t - \frac{f_t^2}{2\theta} \tag{42}$$

$$f_t = R_t - R_{t+1} \in [0, R_t]. \tag{43}$$

h_t represents the FX intervention in period t net of the offsetting effect of the panickers. Unlike in the baseline model, the existence of panickers generates the possibility of counterproductive interventions, which is a phenomenon that EME central banks and FX market commentators have worried about in practice.

Definition 11 (Counterproductive interventions) *FX intervention f_t is counterproductive at the margin if an increase in f_t causes a decrease in h_t. FX intervention is globally counterproductive if $h_t < 0$ for all $f_t > 0$.*

Proposition 9 (Net effect of FX intervention) *In the linear-panickers case (i.e., $\alpha > 0$ and $\theta = \infty$):*

- *Net effect h_t decreases as α increases.*
- *FX intervention is counterproductive both at the margin and globally for $\alpha > 1$.*

In the quadratic-panickers case (i.e., $\alpha = 0$ and $\theta \in \mathbb{R}^{++}$):

- *Net effect h_t, the maximum net effect $H^*(\theta) \equiv \max_{f_t} h_t = \frac{\theta}{2}$, and the level of FX intervention $f_t = \theta$ at which the maximum H^* is achieved, all decrease as θ decreases.*
- *FX intervention is always effective for small levels of intervention, but is counterproductive at the margin for $f_t > \theta$.*

Figure 17. Net Effect of Intervention

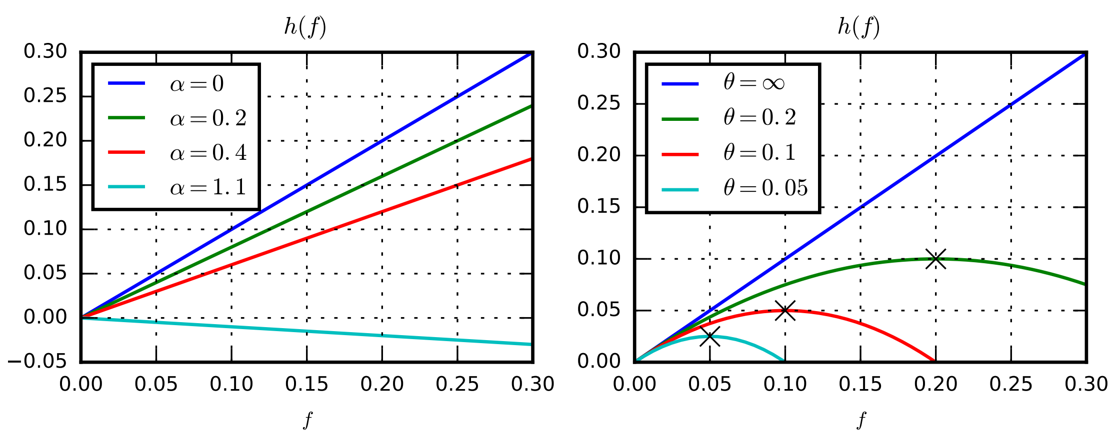


Figure 17 illustrates how h_t varies with f_t. In the linear-panickers case, the net effect of FX intervention diminishes as α increases, and as described in proposition 9, intervention becomes globally counterproductive when $\alpha > 1$. In the quadratic-panickers case, the net effect of FX intervention diminishes as θ decreases, with the maximum net effect $H^*(\theta) = \frac{\theta}{2}$ occurring at lower values of FX intervention $f_t = \theta$. To the right of that peak, FX intervention becomes counterproductive at the margin.

Lemma 7 (Maximum intervention) *An optimizing central bank never chooses a level of FX intervention that is counterproductive (on the margin or globally).*

Theorem 3 (Solution with panickers) *Set $\frac{1}{\beta} = \frac{a+c}{a}$. In the linear-panickers case:*

- $\alpha \in (0,1)$. *The full-commitment FX intervention level is equal to $\frac{\bar{z}}{1-\alpha}$, which is increasing in α, and it stays at this constant value within a non-empty subset of consecutive periods $[t_1, T] \subset [0, \infty)$ that shrinks as α increases. Provided that $\frac{1}{\beta} \leq \frac{(1-\alpha)(a+c)}{(1-\alpha)c-\alpha a}$, the time-consistent FX intervention follows the policy functions $f(R;\alpha)$ and $e(R;\alpha)$, with the following properties:*

$$\frac{d}{d\alpha} e_R(0;\alpha) > 0, \; \frac{d}{d\alpha} f_R(0;\alpha) = 0 \tag{44}$$

$$\lim_{R\to\infty} e(R;\alpha) = e^*, \; \lim_{R\to\infty} f(R;\alpha) = \frac{\bar{z}}{1-\alpha}. \tag{45}$$

- *For $\alpha > 1$, optimal FX intervention $f_t = 0$ irrespective of the degree of commitment.*

In the quadratic-panickers case:

- *The full-commitment FX intervention level is no longer constant during the intervention period $[t_1, T] \subset [0, \infty)$. Provided that $\frac{1}{\beta} \leq \frac{a+c}{c}$, the time-consistent FX intervention follows the policy functions $f(R;\theta)$ and $e(R;\theta)$ with the following properties for small initial reserves:*

$$\frac{d}{d\theta} e_R(0;\theta) = 0, \; \frac{d}{d\theta} f_R(0;\theta) = 0 \tag{46}$$

  For large initial reserves, both the full-commitment and time-consistent FX intervention levels satisfy:

$$\lim_{R_t\to\infty} f_t = \begin{cases} \min\left\{\arg\underset{f_t}{\text{solve}}\,(h_t = \bar{z})\right\} > \bar{z} & \text{for } \theta \geq \overline{\theta}_t \\ \theta & \text{if } \theta < \overline{\theta}_t. \end{cases} \tag{47}$$

$$\lim_{R_t\to\infty} e_t \begin{cases} = e^* & \text{for } \theta \geq \overline{\theta} \\ > e^* & \text{if } \theta < \overline{\theta}, \end{cases} \tag{48}$$

$$\text{where } \overline{\theta} = \arg\underset{\theta}{\text{solve}}\,\{H^*(\theta) = \bar{z}\} = 2\bar{z}. \tag{49}$$

The optimal solution with panickers is presented in the remaining figures of this subsection. We discuss in turn the linear and quadratic cases.

Figure 18. Full Commitment, Various Values of α

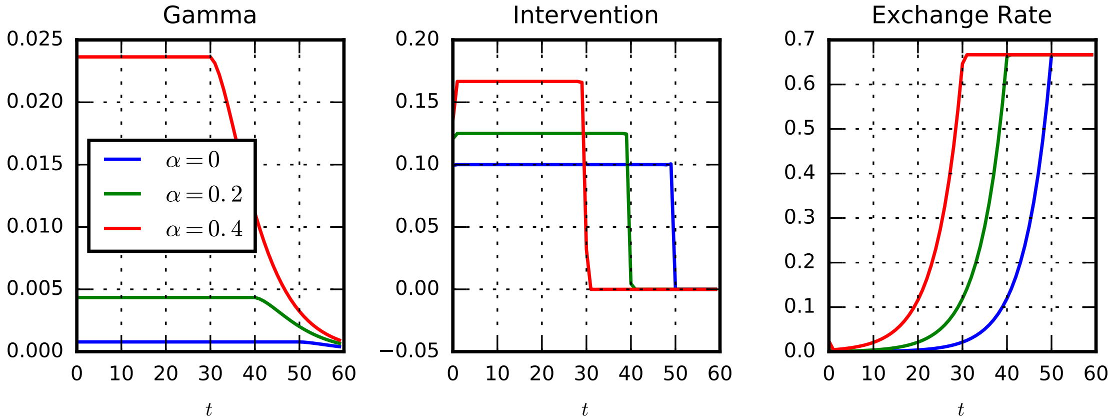


Figure 19. Time Consistent Policy Functions, Various Values of α

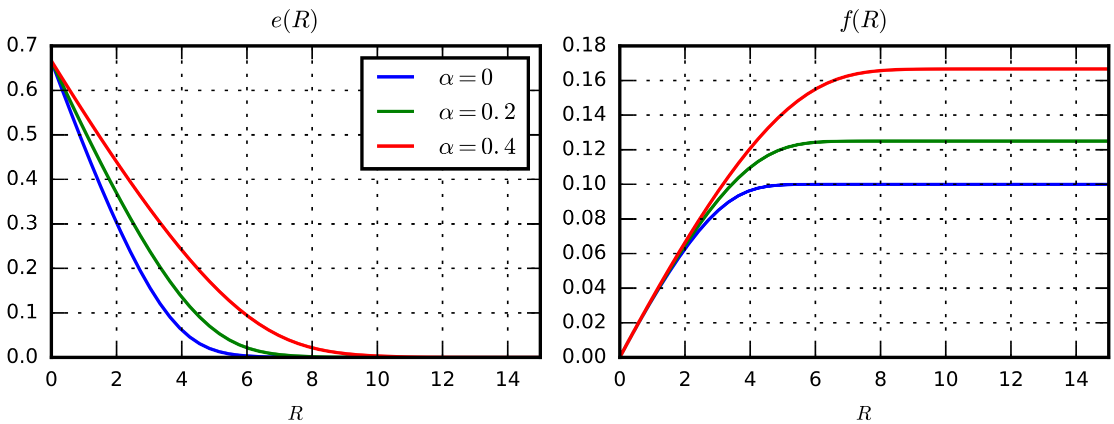


Figure 20. Time Consistent, Various Values of α

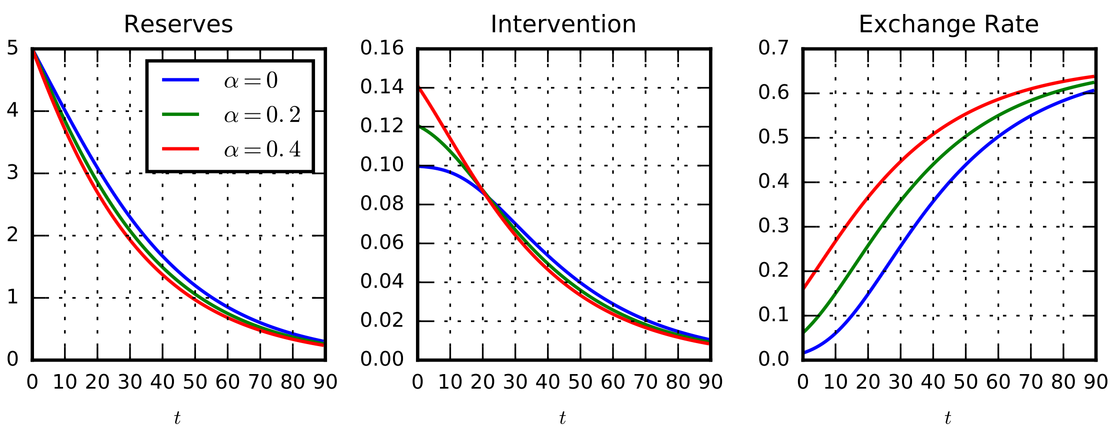


Figure 18 illustrates the full-commitment solution for the linear-panickers case, setting $\frac{1}{\beta} = \frac{a+c}{a}$, $e^* = 0$, $\bar{z} = 0.1$, and $R_0 = 5$. The higher is α, the more severely that panickers reduce the net effect of FX intervention, and in response, there is an "offset effect": the central bank intervenes more heavily than in the baseline case to offset the panicked outflows, $f_t = \frac{\bar{z}}{1-\alpha} > \bar{z}$. More aggressive intervention causes a faster depletion of reserves when $\alpha > 0$ than in the baseline case $\alpha = 0$. The reduced effectiveness of FX intervention means that despite the higher intervention, the exchange rate is stabilized less than in the baseline case.

Figures 19 and 20 illustrate the corresponding time-consistent solutions. In figure 19, the fact that the slope of the FX intervention function $f(R; \alpha)$ is invariant to α close to $R = 0$ shows that for small reserves, no matter how much panic there is, the optimal FX intervention is similar to that in the baseline model solved in subsection 4.2. However, the gentler slope and higher position for the exchange rate function $e(R; \alpha)$ as α increases shows that the presence of panickers reduces the exchange rate stabilization that can be achieved for any level of reserves R. As $R \to \infty$, the time-consistent and full-commitment solutions converge.

Figure 20 shows that starting with a high level of reserves $R_0 = 5$, FX intervention fully offsets the presence of panickers to begin with. The higher is α, the faster that reserves are used up to begin with, followed by slower FX intervention in later periods. The presence of panickers causes poorer exchange rate stabilization in all periods.

In the linear-panickers case, a central bank with a large level of reserves sets FX intervention to fully offset panickers, irrespective of the degree of commitment (the "offset effect"). However, in the quadratic-panickers case, the central bank cannot always fully offset panickers: if it tries to do so, there is a possibility of counterproductive interventions: a deluge of panicked outflows that would actually depreciate the exchange rate. Therefore, the central bank may actually decide to reduce its FX intervention so as to deter the entry of panickers. This phenomenon we call the "deterrence effect."

To explain the quadratic-panickers case, we begin by describing FX intervention at large reserve levels, which are equal irrespective of the degree of commitment, and then we illustrate our findings for lower reserve levels using simulations of optimal time paths. Figure 21 shows how FX intervention at large reserve levels varies with θ, where a reduction in θ corresponds to a higher propensity to panic. For $\theta > \overline{\theta}\,(= 0.2)$, it is feasible to fully offset the impact of the panickers and to keep the net effect of intervention $h_t\,(\leq H^*(\theta))$ equal to the size of the shock $\bar{z}$. Indeed, within this region of θ, a higher propensity to panic induces larger FX intervention to exactly implement such a full offset of the impact of the panickers. Therefore, the "offset effect" prevails, and the exchange rate is perfectly stabilized at e^*.

However, for $\theta < \overline{\theta}$, the propensity to panic is so high that the maximum net effect of intervention $H^*(\theta)$ is smaller than the size of the shock $\bar{z}$. If the central bank intervenes more than θ, then instead of supporting the exchange rate more, panicked outflows would actually depreciate the exchange rate. The central bank finds it optimal to rule out such counterproductive interventions by keeping $f_t \leq \theta$, which deters the excessive entry of panickers. Therefore, the "deterrence effect" prevails, and the consequence of low FX intervention is that the exchange rate is depreciated above e^* even if the central bank has unlimited reserves.

Figure 21. Intervention at Large Reserve Levels

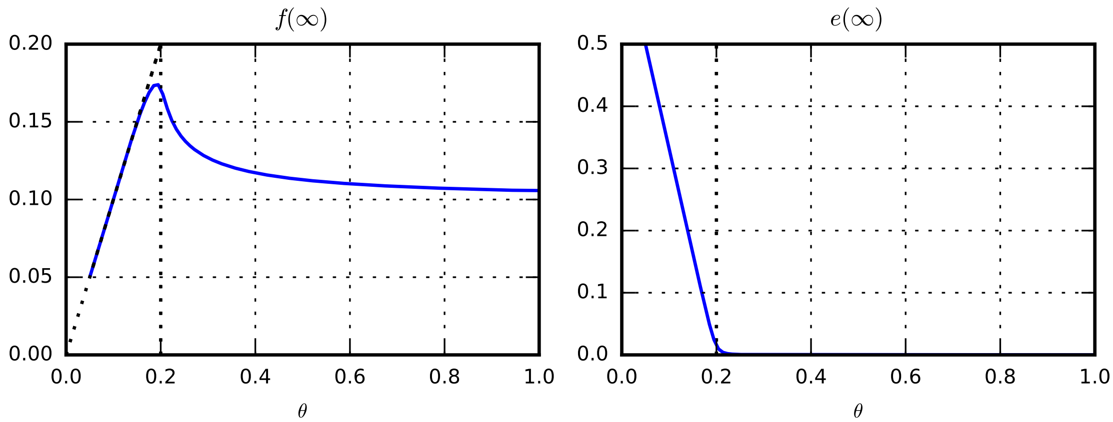


Figure 22 shows the effect of this constraint on exchange rate stabilization in the full-commitment case, for a central bank which starts with a high level of reserves $R_0 = 5$. As θ decreases from ∞ to $\overline{\theta} = 0.2$, the FX intervention at date 0 increases, reflecting the "offset effect." As θ then decreases below $\overline{\theta} = 0.2$, the FX intervention at date 0 decreases one for one, reflecting the "deterrence effect." The central bank becomes unable to fully neutralize the outflow shock $\bar{z}$ despite the large volume of reserves in its vault, and the exchange rate becomes clearly unanchored from the target $e^* = 0$. Even with a large reserves stock and full commitment, for low θ there is a large immediate depreciation as soon as the outflow episode begins.

Figure 23 shows the effect of the constraint on the time-consistent policy functions $e\left(R;\theta\right)$ and $f\left(R;\theta\right)$. The FX intervention function first shifts up as θ decreases from ∞ to $\overline{\theta} = 0.2$ (the "offset effect"), and then down as θ decreases below $\overline{\theta} = 0.2$ (the "deterrence effect"). The time paths for the exchange rate and FX intervention in figure 24 demonstrate that when the propensity to panic is high, the time-consistent solution features a bounded FX intervention level and an exchange rate unanchored from the target $e^* = 0$.

Therefore, when the propensity to panic is high, the full-commitment and time-consistent solutions become closer to each other, featuring a large immediate depreciation irrespective of the level of reserves. Strikingly, both optimal policy responses look less like the full-commitment solution of the baseline model in subsection 4.1, and more like the time-consistent solution in subsection 4.2.

Figure 22. Full Commitment, Various Values of θ

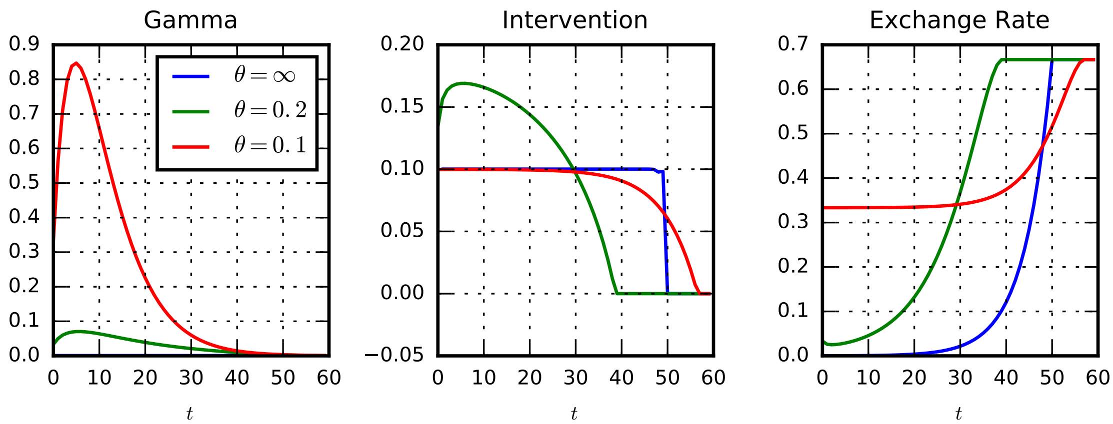


Figure 23. Time Consistent Policy Functions, Various Values of θ

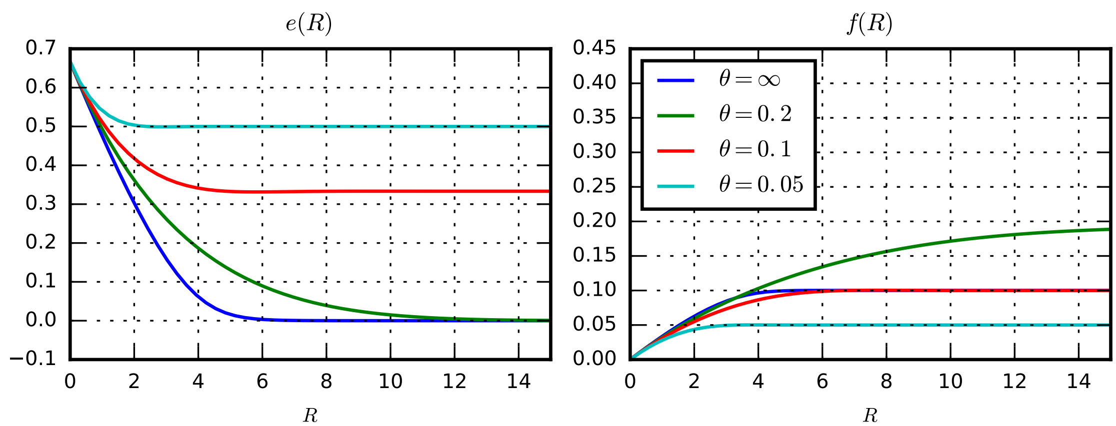
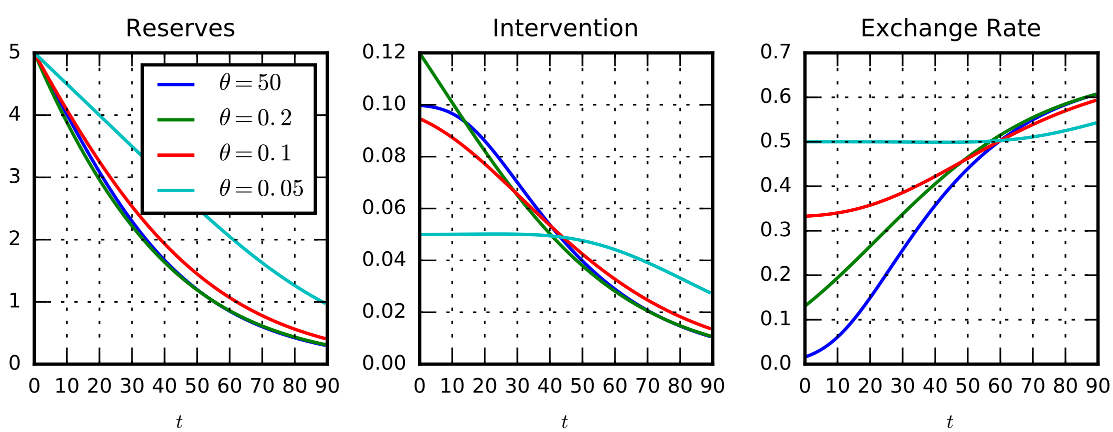


Figure 24. Time Consistent, Various Values of θ



6.2. Knightian flight

While the previous subsection considered a category of foreign investors which removed capital from the domestic economy when reserves declined, let us next consider a qualitatively new category of investors who instead withdraw their investments when the exchange rate depreciates above their comfort level. In particular, consider a new group of unsophisticated "Knightian" investors who lend to the domestic banking system despite their lack of ability to hedge their positions. They abhor even the slightest risk, and their willingness to lend is underpinned by their (misplaced) confidence that the exchange rate will never be allowed to depreciate. In the event that a depreciation does in fact occur, these investors change their expectation of the exchange rate regime, realize that their assets in the domestic economy are risky, and therefore sell all their domestic holdings immediately.

We assume that Knightian investors start off lending L to the domestic banking system, and withdraw all of it in the first period t when $e_t > e^*$. Inspired by central bank actions in actual outflow episodes, we assume that the central bank must provide L in reserves to the banking system during that period in order to ward off a banking system collapse. Despite this support (which exactly cancels the effect of outflows L on the exchange rate), the efficient functioning of the financial markets is still hurt by the withdrawal of a segment of foreign lenders, so in the Knightian flight period, the economy suffers a cost $\Delta = \psi L$.

The baseline model in section 4 is nested within this model by setting the parameters $L = 0$ and $\psi = 0$[31]. It will become clear that we need both of these parameters to be positive to generate interesting effects: the economy must be exposed to Knightian investors and their exit must impose deadweight costs[32].

Using equations (1), (2), (3) and the above information, we can derive the amended set of equations characterizing the feasible set of the model. We assume that $e^* = 0$.

Definition 12 (Model with Knightians) *The reduced-form version of the model is described by the equations for the exchange rate and FX intervention:*

$$e_t = \frac{1}{a+c}\left(\bar{z} - f_t + ae_{t+1}\right) \quad (= 0 \textit{ for } t < T) \tag{50}$$

$$f_t = \begin{cases} R_t - R_{t+1} \in [0, R_t - L] & \textit{for } t < T \\ (R_t - R_{t+1}) - L \in [0, R_t] & \textit{for } t = T \\ R_t - R_{t+1} \in [0, R_t] & \textit{for } t > T, \end{cases} \tag{51}$$

[31]In appendix subsection 8.3, we provide results for the limiting case $L \to 0$ and $\psi L \to \Delta > 0$. This case allows direct comparisons to the exchange rate peg simulations that were solved earlier in subsection 4.3.

[32]The model in this subsection is too simple to capture more sophisticated alternative stories based on information asymmetries, *e.g.*, a setting where the preference of the central bank regarding exchange rate stabilization is unclear, and rational investors study the size of allowed exchange rate depreciations in order to infer the central bank's true preferences. We leave such analysis to future research.

where T is the period in which the peg is broken, and f_t is FX intervention net of central bank support to the banking sector L.

The introduction of Knightian investors partitions the time path $\{e_t, f_t\}_{t=0}^{\infty}$ into two parts: first, during $t \in \{0, 1, ..., T-1\}$, the exchange rate is maintained at the implicit peg of e^*, as the central bank delays the period of Knightian flight; next, during $t \in \{T, T+1, ...\}$, the implicit peg is broken and the exchange rate goes above e^*. For the baseline model $L = 0$ and $\psi = 0$, the central bank never wishes to maintain a peg, so $T = 0$. When $L > 0$ and $\psi > 0$, it may be optimal to choose $T > 0$.

Proposition 10 (Knightians solution) *Let $i = FC$ (full commitment) or TC (time consistent). There exists a period T^i in which the peg breaks because reserves have run out or because the central bank chooses to break the peg before reserves have run out. The exchange rate jumps above $e^* = 0$ from T^i onwards. FX intervention and the peg-breaking time T^i jointly satisfy:*

$$f_t = \begin{cases} \bar{z} & \text{for } t < T-1 \\ \bar{z} + ae^i(R_T) & \text{for } t = T-1 \\ f^i(R_t) & \text{for } t > T \end{cases} \tag{52}$$

$$R_T = R_0 - \bar{z}(T-1) - ae^i(R_T) - L, \tag{53}$$

where:

- *The functions $\{e^{FC}(R_t), f^{FC}(R_t)\}$ and the peg-breaking time T^{FC} are derived from optimization over time-0 welfare under the assumption of commitment.*
- *The functions $\{e^{TC}(R_t), f^{TC}(R_t)\}$ are the baseline time-consistent functions solved in subsection 4.2. The peg-breaking time T^{TC} is the latest time period s in which the following condition is satisfied:*

$$\beta\left[v^{TC}\left(R_s - f_s - L | T^{TC} = s+1\right) - \psi L\right] \geq v^{TC}(R_s - L) - \psi L. \tag{54}$$

We explain the above result step by step. Firstly, in the full-commitment case, the optimal peg-breaking time T^{FC} is chosen to maximize welfare from the perspective of $t = 0$. Figure 25 illustrates the full-commitment solution as ψ increases, setting $\frac{1}{\beta} = \frac{a+c}{a}$, $e^* = 0$, $\bar{z} = 0.1$, $R_0 = 2$ and $L = 0.2$. Notice that with the deterministic outflow shock $\bar{z}$, the central bank knows that it definitely must put aside a quantity L of reserves to support the banks at some date, so irrespective of T^{FC}, it solves the problem with the understanding that L in reserves is unusable for exchange rate support at other times. For $\psi = 0$, the peg is broken immediately, because no welfare is gained from postponing the cost $\Delta = 0$ to the economy, while breaking the peg does provide a benefit because the central bank is able to deliver the welfare-optimizing result of an immediate partial depreciation (which means that it is preserving reserves for use in future periods instead of today). As ψ increases, there is a welfare gain in postponing the cost, so the optimal peg-breaking time T^{FC} increases. Right before the peg breaks, there is a spike in FX intervention to offset the outflows by speculators who know the exchange rate will depreciate next period. The later is T^{FC}, the greater the depreciation when the peg breaks, so the larger the necessary spike in intervention.

Figure 25. Full Commitment, Various Values of ψ

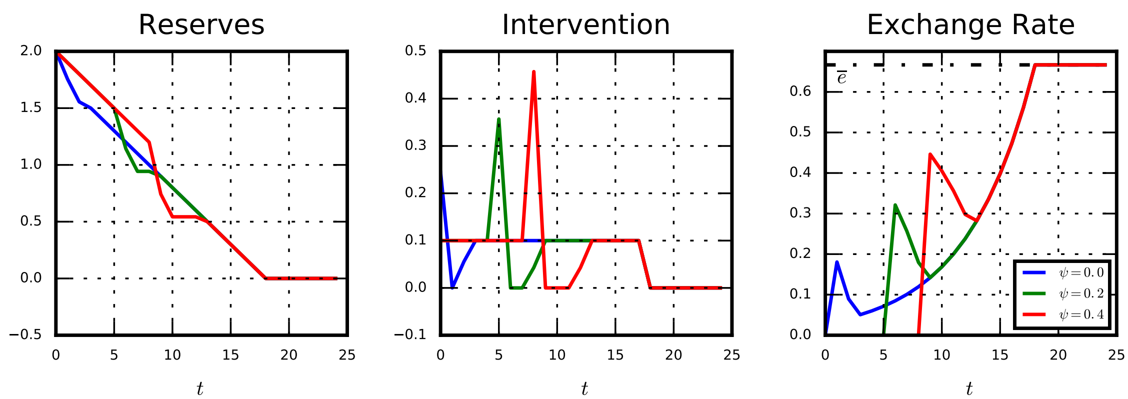
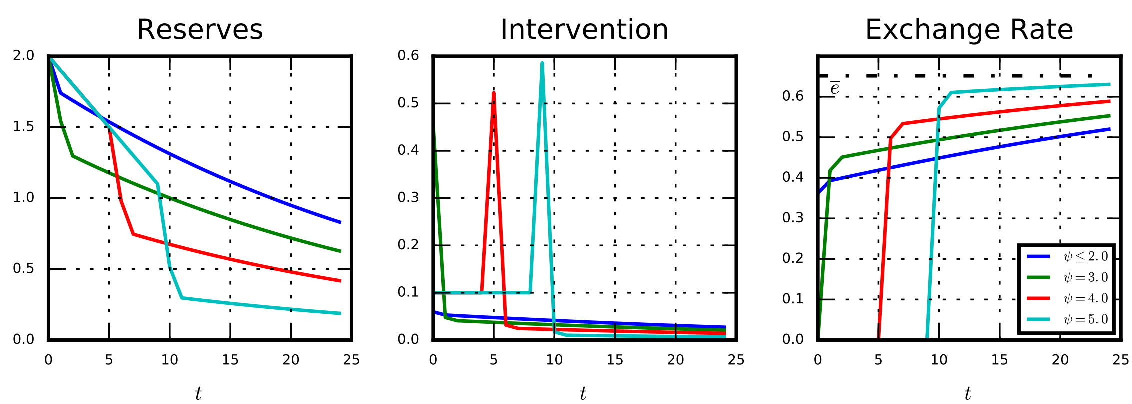


Figure 26. Time Consistent, Various Values of ψ

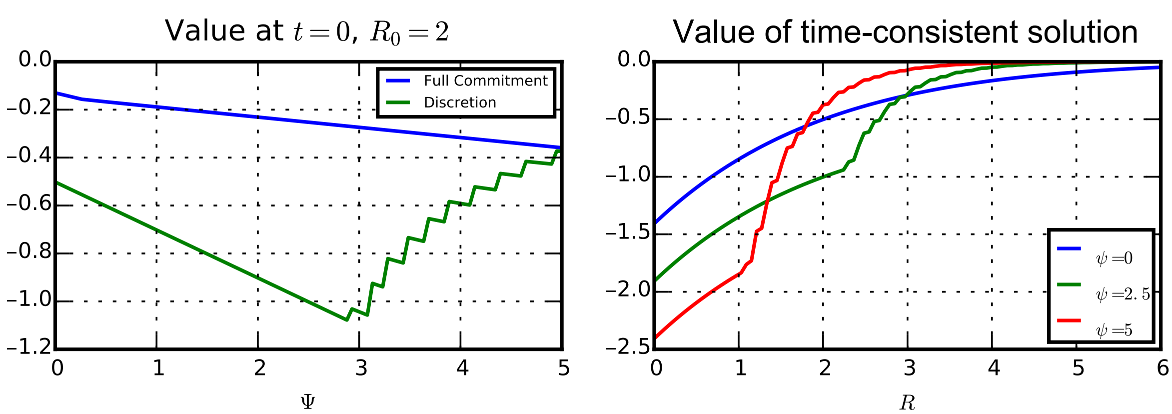


Secondly, in the time-consistent case, the optimal peg-breaking time T^{TC} comes not from an analysis of time-0 welfare, but from an assessment of the temptation in each period to deviate from the continuation of the implicit peg. Equation (54) establishes the trade-off between maintaining the peg for one more period and breaking it today: the left hand side captures the welfare from maintaining the peg in this period, despite the spike in intervention necessary to offset the speculators who know the peg will break in the next period; the right hand side captures the welfare from breaking the peg immediately. For $\psi = 0$, we know that $T^{TC} = 0$, because the left hand side is always smaller than the right hand side: in the baseline time-consistent solution, which yields the $v^{TC}(R_s - L)$ function used in equation (54), maintaining the peg may be a feasible option but never the optimal one. For $\psi > 0$, it may become optimal not to break the peg in this period. The higher is ψ, the later is T^{TC}.

Figure 26 illustrates the time-consistent solution as ψ increases. Since the baseline time-consistent solution features worse exchange rate stabilization than the baseline full-commitment solution, the exchange rate depreciates more when the peg breaks in figure 26 than it does in figure 25. This means that the spike in intervention is larger in the period before the peg breaks. Therefore, the temptation to deviate from the peg and thereby avoid the spike in intervention is high in the time-consistent case, so correspondingly, a high cost ψ is necessary for the peg to be maintained for some periods.

Finally, we assess how the central bank's welfare depends on ψ, the economic cost of Knightian flight per unit of withdrawn loans L. The first panel of figure 27 illustrates the welfare levels under full and zero commitment plotted against ψ, for $R_0 = 2$ and $L = 0.2$. Unsurprisingly, the full-commitment welfare declines as ψ increases. However, the effect of Knightians on the time-consistent welfare is more complicated, because the imperfection of having Knightians in the FX market may offset the imperfection of lack of commitment. For $\psi = 0$, the peg breaks immediately in the time-consistent case. Then as ψ increases to begin with, the time-consistent welfare declines, because the timing of the break is unchanged but the cost of the break is increasing. However, for $\psi \geq 3$, further increases in ψ are able to postpone the breaking of the peg. This postponement prevents the large and welfare-reducing immediate depreciation that occurs in the baseline time-consistent solution, and therefore from the perspective of $t = 0$, the time-consistent welfare actually improves. For every extra period that the peg is maintained, there is an extra step up of the time-consistent welfare, until it reaches the full-commitment level. The time-consistent welfare with $\psi = 5$ is higher than the time-consistent welfare with $\psi = 0$.

Figure 27. Welfare Comparisons with Knightians



The second panel of figure 27 illustrates the impact of ψ on the time-consistent welfare for different values of initial reserves. For small initial reserves, the implicit peg is broken immediately so an increase in ψ just hurts welfare. For large initial reserves, an increase in ψ postpones the breaking of the peg, which improves welfare from the perspective of $t = 0$.

Therefore, a central bank without commitment power, in an economy exposed to Knightian investors with given total loan size L, may benefit from a high cost of exit ψ of these investors. Costly exit hurts the economy in the period of Knightian flight, but provides commitment power to the central bank to implement a temporary peg and thereby avoid a large immediate depreciation.

7. Conclusion

In this paper, we have used a simple stylized framework to tackle the role of FX intervention in the face of outflows in a managed float regime. We have explicitly taken into account the ZLB on reserves, and we have related the optimal policy for an EME central bank to some of the key assessments that it needs to make in every outflow episode—specifically, the level of available reserves, the persistence of the shock and the composition of the FX market. While our optimal solutions for the exchange rate path and FX intervention level depend on the functional forms for welfare and the exchange rate equation of our stylized framework, the qualitative effect of the ZLB on the time consistency of the solution, and on the comparative levels of FX intervention and welfare across different degrees of central bank commitment, should apply across a wide range of models.

We have shown that the level of available reserves is important for the optimal policy decision. For EME central banks which start with a level of reserves that is very large relative to the shock, the optimal FX intervention policy is to fully, or nearly fully, offset the outflow shock and keep the exchange rate stable around the target, irrespective of the degree of commitment. However, when the level of reserves is low or moderate to begin with, the central bank's commitment power becomes important—and central banks which build up their commitment power, through clearly-communicated FX intervention strategies and repeated FX intervention experiences across several outflow episodes, extract significant welfare benefits. After outflow shocks, a central bank with high commitment power can engineer a gradual depreciation to the pure float level, while one with low commitment power can only intervene in a limited manner and must let the exchange rate depreciate nearly to the pure float level. If a central bank with limited reserves does not have an unbounded ability to commit to future policies, but can stick to a pre-announced FX intervention rule, it should commit to such a rule which involves up-front intervention, *e.g.*, a peg or volume intervention rule, to prevent large immediate depreciations once outflow episodes begin.

EME central banks need to assess the persistence of the shock in designing their optimal policies. For temporary shocks, for which the ZLB is expected to bind with only low probability, it is optimal to offset the outflow shock and keep the exchange rate stable around the target, irrespective of the degree of commitment. For persistent shocks, the ZLB is expected to eventually become binding, and the time consistency problem becomes salient. For such shocks, a central bank with high commitment power should promise aggressive intervention in the future rather than today, but a central bank with little commitment power finds itself able to intervene only very little in every period. Therefore, discretionary FX intervention suffices for temporary shocks, while FX intervention rules are valuable for potentially persistent shocks.

Finally, EME central banks should tailor their FX intervention strategies according to the composition of the FX market in which they are intervening. If there exist "panickers" who sell the domestic currency when they observe the central bank's reserves being drawn down, a central bank should offset the extra outflows generated by such investors through aggressive intervention when the propensity to panic is low, but it should be careful to limit intervention and deter the entry of panickers if the propensity to panic is high, despite the destabilization of the exchange rate that ensues. If there exist "Knightian flight" investors who sell all their holdings as soon as the currency depreciates, central banks should maintain an implicit peg for several periods until reserves are drawn down. A central bank with low commitment power may find its effective commitment power increased by the presence of such investors, and in anticipation of this mechanism, it may make sense for such a central bank to cultivate such investors before outflow shocks even begin.

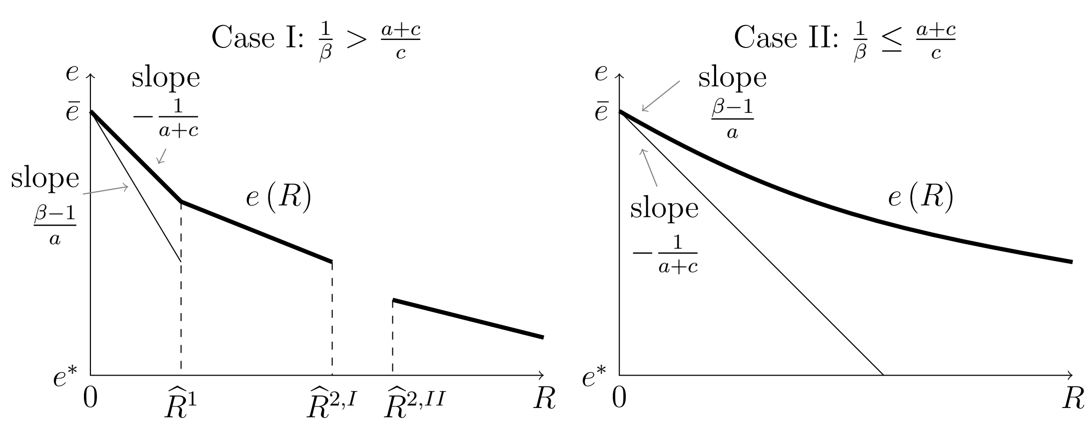
The general message which emerges from our approach is that the characterization of optimal policy in a managed float regime, away from the bipolar extremes of free floats and pegs, is a non-trivial problem. The empirical effectiveness of FX intervention in managing the exchange rate does not immediately imply an obvious FX intervention approach to be adopted by all EME central banks, but rather opens the door to a host of additional considerations such as time consistency, the financial market imperfections which may generate exchange rate shocks, and the heterogeneity of participants within the FX market. Echoing the literature on inflation targeting regimes, the optimal managed float regime requires investment in communication, reputation, and rules. Finally, we expect that the literature should increasingly tailor its recommendations regarding the appropriate managed float regime to the specific market distortions, and associated welfare costs, most salient to each economy and to each central bank's mandate.

8. Appendix

8.1. Proofs of results in the paper

Lemma 1. By inspection. For construction of the feasible set, notice that the most appreciated e_t feasible in period t is achieved by spending all reserves in period t.
Lemma 2. Take the first order conditions (FOC) of the optimization problem (10) with respect to e_t and iterate the formula for Γ_t backward, assuming $e_{-1} = e^*$.
Lemma 3. When $\{f_t\}_{t=0}^{\infty} = \{0\}$, $\{e_t\}_{t=0}^{\infty} = \{\bar{e}\}$ in equation (11), yielding the desired result.
Lemma 4. $\Gamma_t < (a+c)\Lambda$ when f_t is not in the interior of its feasible set. Therefore, intervention should occur when Γ_t is highest according to the lemma 3.
Proposition 1. Take the FOCs of problem (10) with respect to e_t and f_t and rearrange.
Theorem 1. This theorem comes from combining lemma 3 and proposition 1. The hump shape for the marginal value of intervention means that the optimal solution will feature first a set of periods $[0, t_1)$ where Γ_t is increasing, then a set of periods $[t_1, T]$ where Γ_t is constant, then a set of periods (T, ∞) where Γ_t is decreasing. Intervention is not used in periods $[0, t_1)$ because it is more valuable to preserve reserves in these periods, and reserves are completely depleted during periods $[t_1, T]$ because reserves are less valuable in periods after that. The formulae for e_t, f_t, and T follow immediately.
Corollary 1. The envelope condition (EC) establishes that $W_{R_0}^{FC}(R_0) = \Lambda$. From concavity of $W^{FC}(R_0)$, we derive that Λ is decreasing in R_0. Applying equation (12), the desired result follows.
Proposition 2. Take the FOCs of the optimization problem (19) with respect to e and R', and the EC with respect to R, and rearrange.
Theorem 2. <u>Case I:</u> $\frac{1}{\beta} > \frac{a+c}{c}$ (left panel of figure 28).

Figure 28. Proof for Time Consistent Case



First linear segment. For an interior solution at $R = 0$ and $e(0) = \bar{e}$, equation (20) indicates that $e_R(0)$ must be equal to $\frac{\beta-1}{a} < -\frac{1}{a+c}$, which violates the feasibility constraint derived in lemma 1. Therefore, the constraint is binding at $R = 0$, as the central bank wishes to spend more than 0. Assuming continuous differentiability of $e(R)$ within a neighborhood of $R = 0$, we see that the constraint remains binding for R within some region $\left[0, \widehat{R}^1\right]$; within this region, feasibility and the generalized Euler condition together mean that $e_R(R) = -\frac{1}{a+c}$. $\widehat{R}^1$ satisfies the condition that when equations (20) and (21) are evaluated with $e_R(R) = -\frac{1}{a+c}$, we obtain $\widehat{R}^1 - f\left(\widehat{R}^1\right) = 0$. Notice that $e\left(\widehat{R}^1\right) > e^*$.

Second linear segment. To the right of $\widehat{R}^1$, the feasibility constraint is no longer binding and the solution for $f(R)$ becomes interior. We consider reserve levels R such that $f(R) < R$, $R - f(R) \in \left(0, \widehat{R}^1\right]$, and $e_R(R - f(R)) = -\frac{1}{a+c}$. From equations (20) and (21), the slope $e_R(R)$ to the right of $\widehat{R}^1$ must again be constant:

$$e_R(R) = -\frac{\beta \frac{1}{a+c}}{\beta + \left(\frac{c}{a+c}\right)^2}, \tag{55}$$

which is less negative than $-\frac{1}{a+c}$. The $e(R)$ function does not jump vertically at $\widehat{R}^1$ because $\widehat{R}^1$ itself is also defined using the slope $e_R(R - f(R)) = -\frac{1}{a+c}$ in equations (20) and (21). Therefore, there exists some region $\left[\widehat{R}^1, \widehat{R}^{2,I}\right]$ for which $e(R)$ has a kink but no jump at $\widehat{R}^1$, with $\widehat{R}^{2,I}$ satisfying $\widehat{R}^{2,I} - f\left(\widehat{R}^{2,I}\right) = \widehat{R}^1$ and $e_R\left(\widehat{R}^{2,I} - f\left(\widehat{R}^{2,I}\right)\right) = -\frac{1}{a+c}$. Notice that $e\left(\widehat{R}^{2,I}\right) > e^*$.

Third linear segment. For $R < \widehat{R}^{2,I}$, we have $R - f(R) < \widehat{R}^1$, so $e_R(R - f(R)) = -\frac{1}{a+c}$, but for $R > \widehat{R}^{2,I}$, we have $R - f(R) > \widehat{R}^1$, so $e_R(R - f(R)) = -\frac{\beta \frac{1}{a+c}}{\beta + \left(\frac{c}{a+c}\right)^2} > -\frac{1}{a+c}$. From equations (20) and (21), if $\widehat{R}^{2,I} - f\left(\widehat{R}^{2,I}\right) = \widehat{R}_1$ with $e_R\left(\widehat{R}^{2,I} - f\left(\widehat{R}^{2,I}\right)\right) = -\frac{1}{a+c}$, then there exists some $\widehat{R}^{2,II} > \widehat{R}^{2,I}$ and $e\left(\widehat{R}^{2,II}\right) > e^*$ satisfying $\widehat{R}^{2,II} - f\left(\widehat{R}^{2,II}\right) = \widehat{R}_1$ and $e_R\left(\widehat{R}^{2,II} - f\left(\widehat{R}^{2,II}\right)\right) > -\frac{1}{a+c}$:

$$\left(\widehat{R}^{2,I} - \widehat{R}^1\right) = \bar{z} - (a+c)\frac{\beta e\left(\widehat{R}^1\right)}{1 + a \lim\limits_{R \to \left(\widehat{R}^1\right)^-} e_R\left(\widehat{R}^1\right)} + ae\left(\widehat{R}^1\right)$$

$$< \left(\widehat{R}^{2,II} - \widehat{R}^1\right) = \bar{z} - (a+c)\frac{\beta e\left(\widehat{R}^1\right)}{1 + a \lim\limits_{R \to \left(\widehat{R}^1\right)^+} e_R\left(\widehat{R}^1\right)} + ae\left(\widehat{R}^1\right). \tag{56}$$

Therefore, the $e(R)$ function has gaps: it is not defined in the region $\left(\widehat{R}^{2,I}, \widehat{R}^{2,II}\right)$ when we restrict ourselves to using the different left and right derivatives at $\widehat{R}^1$ when construct the mapping from $\widehat{R}^{2,I}$ and $\widehat{R}^{2,II}$ to $\widehat{R}^1$. The $e(R)$ function can be "filled in" within the region $\left(\widehat{R}^{2,I}, \widehat{R}^{2,II}\right)$ if we allow unrestricted hypothetical derivatives at $\widehat{R}^1$ (i.e., unrestricted off-equilibrium beliefs), but this assumption does not satisfy subgame-perfection: if $f(R)$ actually deviates from the optimum, $e(R - f(R))$ deviates according to either the left or the right derivative at $R - f(R)$, not any other value of the derivative.

Non-existence. Assuming continuous differentiability in a neighborhood of $R = 0$, the only possible subgame-perfect solution must be piecewise-linear with kinks and gaps. Kinks mean that continuous differentiability is violated for the $e(R)$ function, which is undesirable but not necessarily disqualifying per se. However, kinks eventually lead to gaps in the $e(R)$ function—so for some values of R, the optimal policy does not exist. We rule out such solutions. So we rule out solutions for $\frac{1}{\beta} > \frac{a+c}{c}$.

<u>Case II:</u> $\frac{1}{\beta} \leq \frac{a+c}{c}$ (right panel of figure 28). The feasibility constraint is not violated at $R = 0$, so a solution may exist. The desired results follow from equations (20) and (21).

Corollary 2. For $\frac{1}{\beta} \leq \frac{a+c}{c}$, any time-consistent solution features $f(R) < R$, which yields the result.

Proposition 3. This result follows from the exchange rate equation and definition 6.

Proposition 4. This result follows from the exchange rate equation and definition 7.

Proposition 5. The first two bullets are proved by inspection. The third bullet is proved by construction (see figure 11).

Lemma 5. By inspection.

Proposition 6. Take the FOCs of the optimization problem (32) with respect to $e_t(s^t)$ and $f_t(s^t)$ and rearrange.

Proposition 7. Take the FOCs of the optimization problem (36) with respect to $e(R, \bar{z})$ and $R'(R, \bar{z})$, and the envelope condition (EC) with respect to R, and rearrange.

Corollary 3. For $\frac{1}{\beta} \leq \frac{(a+c)p}{a(1-p)+c}$, any time-consistent solution features $f(R, \bar{z}) < R$ by analogy to the deterministic model. The desired result follows.

Proposition 8. The result for $p = 1$ is proved by construction. As p decreases below 1, $e(0, \bar{z}) = \frac{z}{a(1-p)+c}$ decreases and $e'(0, \bar{z}) = \frac{\beta p - 1}{ap}$ becomes steeper, both of which increase the range of reserve values for which the time-consistent intervention level is set equal to z, as it is in the full-commitment case. The time-consistent solution no longer exists once p declines so much that $\frac{1}{\beta} > \frac{(a+c)p}{a(1-p)+c} \Leftrightarrow p < \frac{1}{\beta}\frac{a(1-p)+c}{(a+c)}$.

Lemma 6. Suppose that $z_t(s^t) = \bar{z}$. Use equation (35) to consider allocations from period t onwards, working backwards from the future to the present. Firstly, assuming that $z_{t+1}(s^{t+1}) = 0$, $\{e_u(s^u)\}_{u=t+2}^{\infty}$ diverges above e^* forever if $e_{t+2}(s^{t+2}) > e^*$, which is not optimal, and diverges below e^* forever if $e_{t+2}(s^{t+2}) < e^*$, which is neither optimal nor feasible. Therefore, at the optimum, if $z_{t+1}(s^{t+1}) = 0$, then $\{e_u(s^u)\}_{u=t+2}^{\infty} = \{e^*\}$ and $\{f_u(s^u)\}_{u=t+2}^{\infty} = \{0\}$. Secondly, if $e_{t+1}(s^{t+1})$ takes a positive value for $z_{t+1}(s^{t+1}) = \bar{z}$, as is necessary, it must take a negative value for $z_{t+1}\left(s^{t+1}\right)=0$. The only way that the latter can be achieved is through $f_{t+1}\left(s^{t+1}\right)>0$ for $z_{t+1}\left(s^{t+1}\right)=0$.

Proposition 9. By inspection.

Lemma 7. By definition of optimality.

Theorem 3. For both the linear and quadratic cases, irrespective of the degree of commitment, intervention fully stabilizes the exchange rate for large R provided that $h(f)$ can be at least as high as $\bar{z}$; if the latter condition is not satisfied, then consistent with lemma 7, intervention is set to achieve the maximum exchange rate stabilization possible for large R. This argument yields the desired results for large R. Regarding the time-consistent policy, interior solutions satisfy:

$$\left(e\left(R\right)-e^{*}\right)\left[h_{f}\left(f\left(R\right)\right)+ae_{R}\left(R-f\left(R\right)\right)\right]=\beta\left(e\left(R-f\left(R\right)\right)-e^{*}\right)h_{f}\left(f\left(R-f\left(R\right)\right)\right) \tag{57}$$

$$e\left(R\right)=\frac{1}{a+c}\left[\overline{z}-h\left(f\left(R\right)\right)+ae\left(R-f\left(R\right)\right)\right] \tag{58}$$

$$h\left(f\right)=\left(1-\alpha\right)f-\frac{f^{2}}{2\theta}. \tag{59}$$

Taking the limit as $R\rightarrow 0$ in equation (57) and then differentiating equations (58) and (59), the desired results follow:

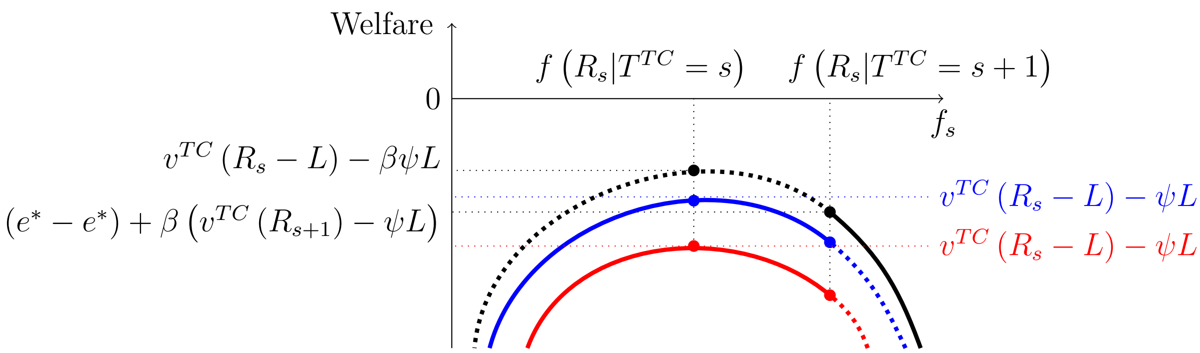
$$e_{R}\left(0;\alpha,\theta\right)=\frac{\left(\beta-1\right)\left(1-\alpha\right)}{a} \tag{60}$$

$$f_{R}\left(0;\alpha,\theta\right)=\frac{1-\beta}{\beta}\frac{c}{a}. \tag{61}$$

The condition $\frac{1}{\beta}\leq\frac{(1-\alpha)(a+c)}{(1-\alpha)c-\alpha a}$ allows for the existence of the time-consistent solution.

Proposition 10. By inspection. Figure 29 illustrates the peg-breaking decision from equation (54). The black line plots the welfare given that the peg breaks at $T^{TC}=s+1$. The blue and red lines represent welfare given that the peg breaks at $T^{TC}=s$.

Figure 29. Incentive to Maintain the Implicit Peg



The relative orientation of the black and blue lines is what the comparison from equation

(54) looks like when ψ is low. The relative orientation of the black and red lines is what the comparison from equation (54) looks like when ψ is high. The higher is ψ, the greater the incentive to maintain the peg.

8.2. Taylor series expansions

We present the Taylor series expansion of $f(R)$ and $e(R)$ at $R = 0$ using equations (20) and (21) from the deterministic case. As the order of the Taylor series increases, there appears to be a radius of convergence in the neighborhood of $R = 0$, but not for large R.

Figure 30. First Order Taylor Expansion

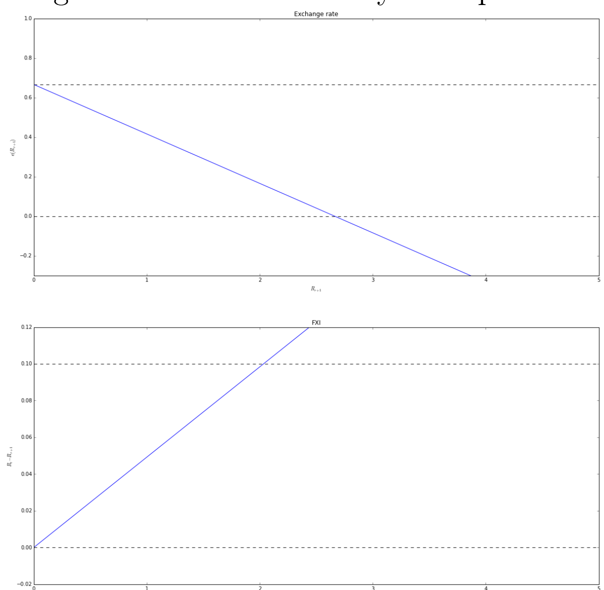


Figure 31. Second Order Taylor Expansion

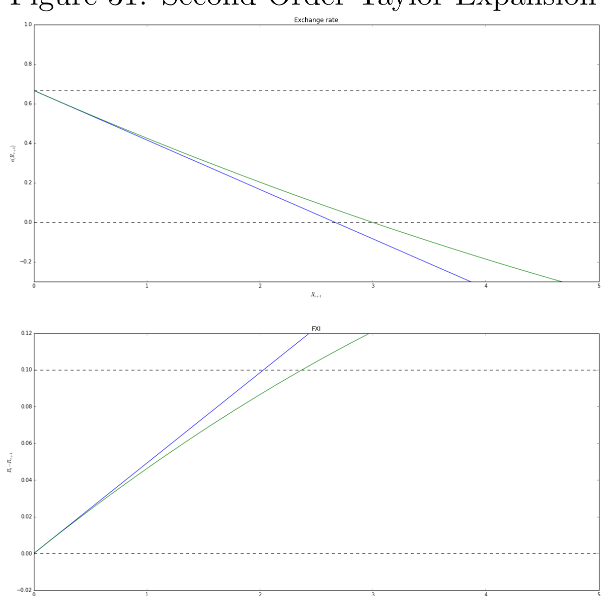


Figure 32. Third Order Taylor Expansion

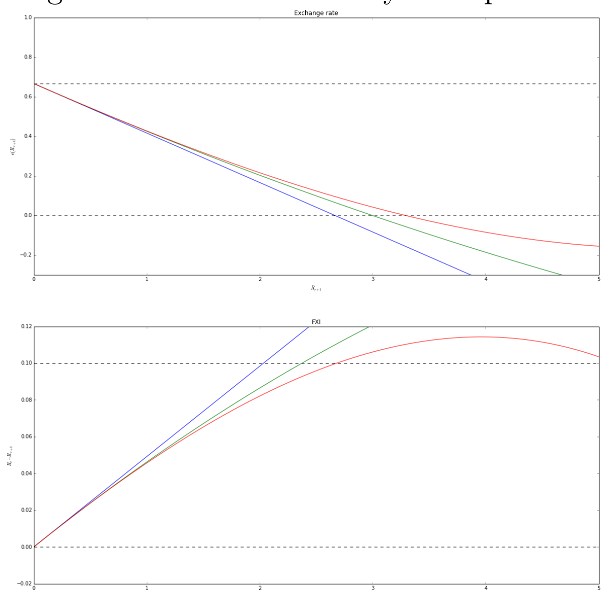
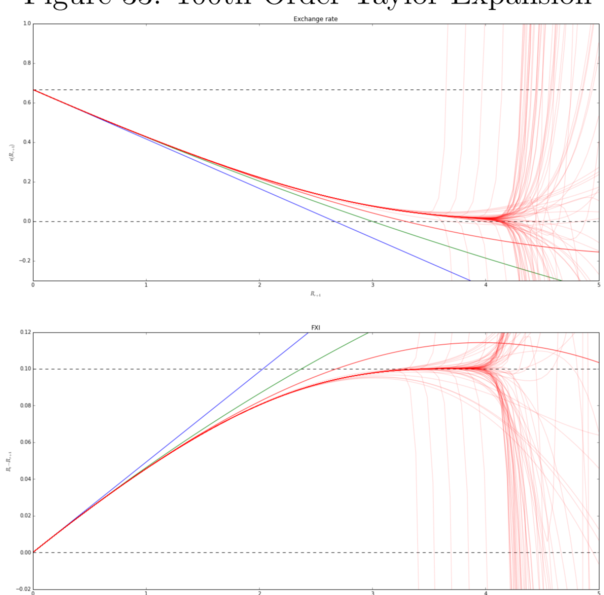


Figure 33. 100th Order Taylor Expansion



8.3. Knightians in the limiting case

Here we present the time-consistent welfare level for the limiting case of the model with Knightians: $L \to 0$ and $\psi L \to \Delta > 0$. The purpose of this exercise is to establish a more direct comparison between the model with Knightians in subsection 6.2 and the baseline solutions in section 4: for this limiting case, we can ignore the effect of L so the model is identical to the baseline one except that there is a cost Δ of breaking the peg.

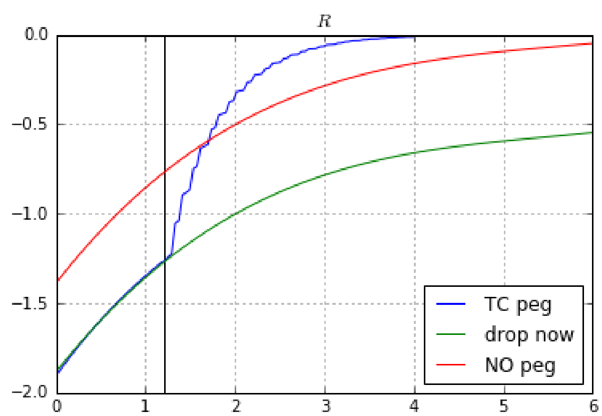
The solution to this limiting case satisfies proposition 10, but with an amended version of equation (54):

$$\beta\left[v^{TC}\left(R_s - f_s | T^{TC} = s+1\right) - \Delta\right] \geq v^{TC}\left(R_s\right) - \Delta. \tag{62}$$

Can the introduction of $\Delta > 0$ improve time-consistent welfare relative to the baseline model with $L = 0$ and $\Delta = 0$? The answer is yes. Figure 34 illustrates the time-consistent welfare as a function of initial reserves R. When $L = 0$ and $\Delta = 0$, the peg breaks immediately and the time-consistent welfare is given by the red line. If we set $\Delta > 0$ and force the peg to still break at $t = 0$, then the time-consistent welfare shifts to the green line. If we set $\Delta > 0$ and allow the peg-breaking time to adjust according to equation (62), then the new time-consistent welfare is given by the blue line.

Therefore, for this limiting case of the model with Knightians, a central bank with large initial reserves can improve on the time-consistent welfare calculated in subsection 4.2. For small initial reserves, the implicit peg is broken immediately so an increase in Δ just hurts welfare. For large initial reserves, an increase in Δ postpones the breaking of the peg, which improves welfare from the perspective of $t = 0$.

Figure 34. Time Consistent, Cost Δ of Breaking the Peg



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