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Equity Depletion from Government-Guaranteed Debt

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Abstract

Government guarantees of private debt deplete equity. The depletion is greatest during periods when the probability of a guarantee payoff is highest. In a setting otherwise subject to Modigliani-Miller neutrality, firms issue guaranteed debt up to the limit the government permits. Declines in asset values raise debt in relation to asset values and thus deplete equity directly, under the realistic assumption that the government is unable to enforce rules calling for marking asset values to market. Less widely recognized is that guaranteed debt creates an incentive to pay equity out to owners—such a payout lowers the value of the firm's call option on its assets by less than the amount of the payout. I build a simple dynamic equilibrium model of an economy that would follow a smooth growth path but for the existence of debt guarantees. Exogenous changes in asset prices cause major swings in allocations as participants respond to changing incentives. Periods when default is unusually likely because asset prices have fallen are times of abnormally high consumption. The withdrawal of equity from firms to pay for the consumption will turn out to be free on the margin if the firm defaults. Consumers concentrate their consumption during the periods when consumption is cheap. Because these periods are transitory, they generate expectations of negative consumption growth, which implies that interest rates are low. Thus guarantees generate substantial volatility throughout the economy.

*This research is part of the program on Economic Fluctuations and Growth of the NBER. A file containing the calculations is available at my website.

1 Introduction

In modern economies, the government guarantees the debt of many borrowers. In a few cases, the promise is explicit; in others it is implicit but known to be likely; and in others, the guarantee occurs because the alternative is immediate collapse, with substantial harm to the rest of the economy. The modern government cannot stop itself from making good on the obligations of many borrowers, large and small. I demonstrate that debt guarantees deplete equity from firms at times of asset-price declines. Not only do firms fail to replace equity lost when leveraged portfolios lose value, but they have an incentive to deplete equity further, by paying unusually high dividends.

The government adopts a safeguard to protect the taxpayers against the worst abuses of guarantees—it imposes a capital requirement to limit the ratio of guaranteed debt to the value of the underlying collateral.

In the United States, organizations with explicit guarantees on some debt (deposits) are mainly banks. The non-deposit obligations of banks and other intermediaries, notably the two huge mortgage-holders, Fannie Mae and Freddie Mac, enjoy market values that only make sense on the expectation of a government guarantee. The recent intervention to avert the collapse of Bear Stearns confirmed that the government will pay off on private debt obligations in times of stress even in sectors distant from any formal debt guarantees. The Federal Housing Administration guarantees the debt of individual mortgage borrowers; the government is extending this guarantee to a larger set of individual borrowers under pressure from declining housing prices. Almost any borrower faces some probability that adverse future events will result in the government repaying the borrower's debt.

A key issue is the withdrawal of equity capital from firms with guaranteed debt—the phenomenon I call *equity depletion*. A counterpart is the unwillingness of investors to supply equity to firms that face positive probabilities of insolvency and payoffs on government guarantees. Equity depletion rises along with the probability of default rises. Withdrawing equity from a firm in one period has zero marginal cost in states next period where default occurs and the government pays off on guaranteed debt—the value of equity claims is zero in those states. If a larger fraction of future states have zero equity value, the expected payoff to equity investments this period declines. In a partial equilibrium setting, this factor would result in knife-edge behavior—the owners of a firm would pay out all of the equity value of a firm as a dividend so as to maximize the value of the government bailout. Akerlof and Romer (1993) describe actions of this type leading up to the savings and loan crisis in the U.S. in the 1980s. In the model of this paper, however, a countervailing force limits the depletion of capital. Any injection of equity to firms in general comes from reduced consumption and any removal of capital takes the form of a consumption binge. With a non-zero value of the elasticity of marginal utility with respect to consumption, the desire to smooth consumption keeps equity flows into and out of firms at finite rates. But consumption does rise substantially in periods when expected defaults and accompanying bailouts become more likely.

The paper reaches these conclusions in a simple general-equilibrium model. Capital is the only factor of production; output is proportional to capital. The marginal product of capital is a constant and so is the economy's real return, absent any distortions from guaranteed debt. Each period, consumers decide between consuming and saving. The economy grows at a constant rate—it has no natural source of volatility. To this economy I add a monetary system that generates an exogenous rate of inflation or deflation in the price level. The government guarantees nominal debt secured by capital.

I characterize the limitations on the government's debt guarantee in what I believe is a realistic way. The government enforces a capital requirement: At the time a company issues debt, the amount borrowed may not exceed a specified fraction of the value of the capital. The remaining value is the borrower's equity, sufficient to meet the capital requirement. If the price level falls enough so that the new value of the capital falls short of the value of the debt, the government makes up the difference. The lender receives a payment of the difference. Equity shareholders in the firm receive nothing back when default occurs and the government pays off.

My characterization of the government's capital requirement has an important dynamic element: If the price level falls, but not enough to push the borrower into insolvency, the borrower may keep nominal debt at its earlier level. The government fails to follow the principle of prompt corrective action. Under that principle, the borrower would mark its collateral to market and could borrow only the specified fraction of the new, lower value of the collateral. Instead, the government acts as if the collateral had its historical nominal value and permits the borrower to keep the historical level of debt, which the government guarantees.

Figure 1 shows the operation of the model in an example of 50 years of experience. The upper panel shows the price level. It drifts upward, with occasional reversals. Its time-series properties mimic those of the nominal value of the S&P 500 stock-market index. The price level is the exogenous driving force of the model, the only source of departures from a smooth growth path. The upper panel also shows a key variable of the model, the fraction of the value of capital financed by equity. In periods when the price level is constant or rising, the fraction is about 30 percent, reflecting the government's capital requirement of that amount. But when the price level falls, equity depletion occurs—the equity fraction declines. One reason is the government's rule that firms may keep debt at its previous nominal level even when deflation occurs. Capital requirements are based on the book value of assets, not the lower market value. But another reason is the incentive for firms to increase their payouts when default looms.

In the 50-year history shown in the figure, default occurs twice. In the first one, three consecutive price declines cause moderate equity depletion, followed by default. In the second default, four consecutive shocks deplete equity essentially entirely, leaving firms on the brink of default. A fifth decline causes default with a large guarantee payoff. At the end of the 50 years, a group of deflationary shocks depresses equity to about 20 percent of capital but does not cause default.

The lower panel of Figure 1 shows the resulting volatility in real consumption and in the nominal interest rate. Consumption rises whenever equity depletion occurs from deflation. Consumption remains high as long as equity is positive and below normal. Consumers perceive that extracting equity from firms and consuming it may be free in a year when default is unusually likely, because the increased guarantee pays for the consumption should default occur. Consumption is strongly mean-reverting in this economy. When consumption is high, it will probably decline next year either because an inflationary shock returns equity to normal or because a deflationary shock will cause default, in which case consumption also returns to normal. Thus periods of depleted equity are also periods of negative expected consumption growth. The model has constant expected inflation, so the main factor causing the interest rate to change is expected consumption growth. As the figure shows, the nominal interest rate falls dramatically during periods of depleted equity. The model permits negative nominal rates because its monetary sector has no explicit monetary instrument.

It should go without saying that the model in this paper falls short of capturing reality. It makes no claim to portray the actual events in financial markets in 2007 and 2008. Rather,

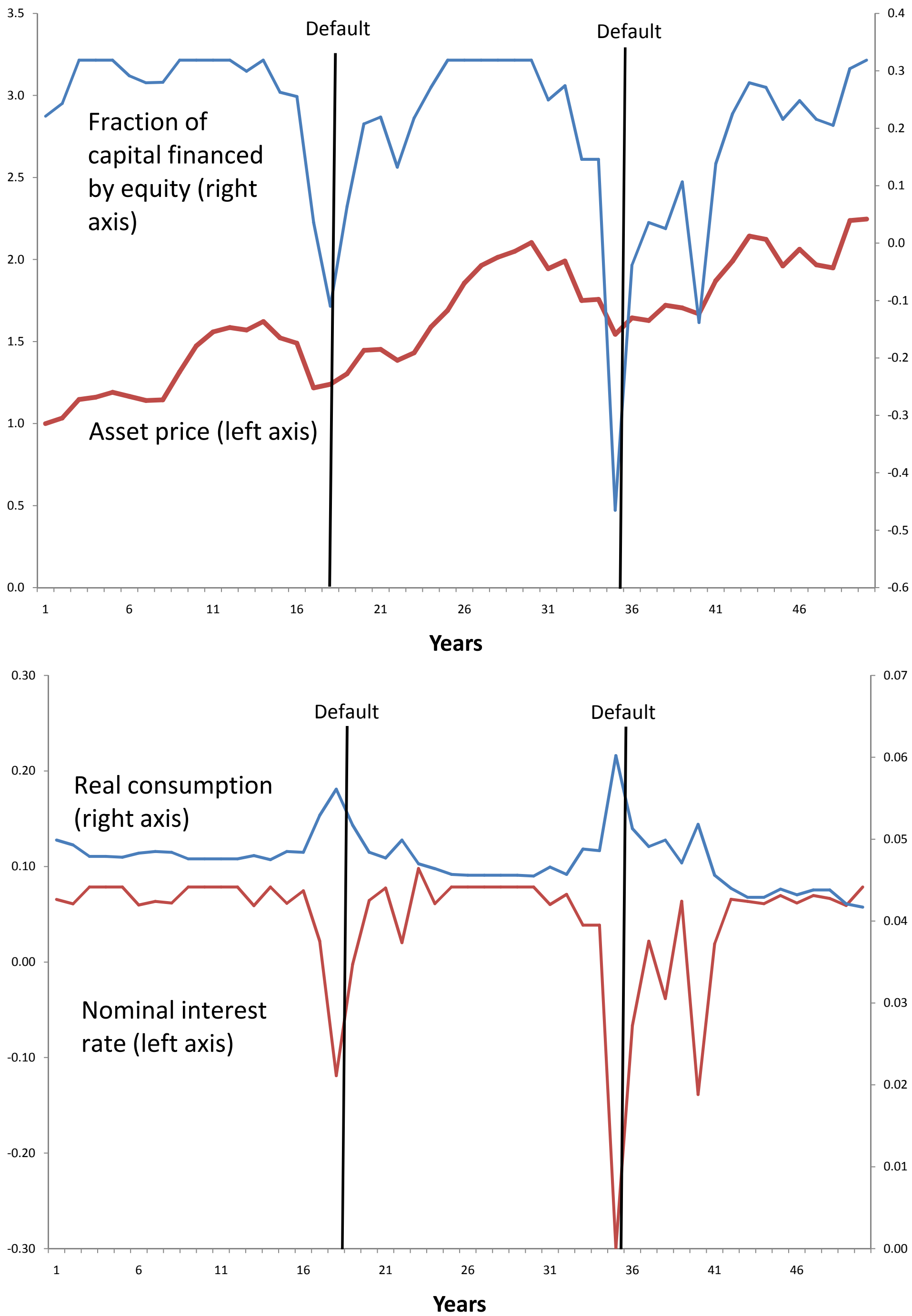

Figure 1: Example of a History From the Model

it is a full working out of the implications in a fairly standard model of one important feature of financial markets, widespread government guarantees of debt.

2 Options

Black and Scholes (1973) identified an incentive for equity depletion for a firm with non-guaranteed debt. In that situation, the shareholders hold a call option on the firm's assets with an exercise price equal to the face value of the debt. If the firm depletes equity by, for example, paying a dividend of $1, the shareholders gain $1 in hand but lose less than $1. The δ of the option—the derivative of its value with respect to the value of the firm's assets—is less than one. Hence it would appear that the shareholders have an unlimited incentive to deplete equity. But every dollar the shareholders gain is a dollar lost by the bondholders. Black and Scholes went on to observe that bondholders have contractual protections against this conduct.

Equity depletion without a debt guarantee is the result of exploitation of bondholders by shareholders. If investors all held shares and bonds in the same proportion, the incentive for equity depletion would not exist. Equity depletion arising from the incentive identified by Black and Scholes is the result of agency frictions between shareholders and bondholders. This paper is *not* about that incentive. I assume a frictionless solution to the potential agency problem.

With guaranteed debt, the Black-Scholes analysis continues to hold—paying out $1 in dividends lowers the value of the shareholders' call option by less than $1. The guarantee immunizes the bondholders from any loss, however. The shareholders capture extra value not from the bondholders but from the government. Equity depletion appears to be an unlimited opportunity to steal from the government.

Just as the bondholders have at least some contractual protection in the case of non-guaranteed debt, the government, in the model developed here, imposes restrictions on equity depletion. In particular, firms cannot steal directly by paying out such high dividends that debt exceeds the current value of assets. Firms may only gamble that a decline in asset values next period will trigger a guarantee payment. But it turns out that this constraint rarely binds. In the aggregate, equity depletion requires increased consumption. With a reasonable value of the elasticity of intertemporal substitution, consumers' desire to smooth consumption limits equity depletion, though equilibrium consumption in the model is quite volatile.

3 Model

3.1 Basic growth model

Capital K is the only factor of production. One unit of capital becomes $1+r$ units of output at the beginning of the next period. Each person consumes c. I let $'$ (prime) denote next period's value of a variable. I denote state variables with a $\hat{}$. The state variable associated with capital is the quantity of output available for reinvestment or consumption, $\hat{K}$. Its law of motion is

$$\hat{K}' = (1+r)(\hat{K} - c) \tag{1}$$

Consumers have power utility functions with coefficient of relative risk aversion of γ and intertemporal elasticity of substitution of $1/\gamma$. Consumers solve the dynamic program,

$$V(\hat{K}) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + \beta V(\hat{K}'). \tag{2}$$

Here β is the consumer's discount ratio. The value function takes the form

$$V(\hat{K}) = V\hat{K}^{1-\gamma}. \tag{3}$$

so the dynamic program becomes

$$V\hat{K}^{1-\gamma} = \max_c \frac{c^{1-\gamma}}{1-\gamma} + \beta V\hat{K}'^{1-\gamma}. \tag{4}$$

The solution for V and c does not have a closed form but is unique and computationally benign. Notice that one could divide both sides by $\hat{K}^{1-\gamma}$ to eliminate capital as a state variable, replacing c with $\tilde{c} = c/\hat{K}$. I take this approach in the computations for this paper but do not use it in the exposition.

The economy will grow if the productivity of capital exceeds the impatience of consumers, in the sense that $(1+r)\beta > 1$. For the rest of the paper, I will consider the stationary case, where $(1+r)\beta = 1$. In this economy, consumption per unit of capital is the amount needed to keep capital constant:

$$c = \frac{r}{1+r}\hat{K}. \tag{5}$$

Nothing ever happens in this economy.

3.2 Firms and consumers

The basic growth model has constant returns to scale, so the boundaries of the firm are indeterminate. Without loss of generality, I assume that each consumer has an equity interest in one firm. The consumer's claims on the firm are a mixture of a one-period nominal debt claims and a residual equity claim. The consumer makes all decisions for the firm, including the level of capital and the amount of debt. In the absence of government guarantees on the debt, the firm satisfies the Modigliani-Miller property of indifference to the mix of debt and equity.

3.3 Government

The government intervenes in the economy in two ways. First, it establishes a monetary unit and manages it so that the price of one unit of goods is p. I picture p as an asset price, but it is also the price of consumption. The change ratio p'/p is exogenously, independently, and identically distributed over time. Uncertainty about the change in the asset price is the only source of aggregate uncertainty in the economy. The model focuses on deflation because a decline in asset values triggers the interesting events in the economy. If deflation never occurs, the static equilibrium of the basic growth model prevails at all times.

Second, the government guarantees nominal debt secured by capital. If the price level next period falls enough to make the nominal value of the capital less than the nominal debt, the government pays the shortfall. The payout is the difference between the amount of debt and the nominal value of the collateral capital—the usual amount of a government payment for defaulted guaranteed debt or the cost of a bailout. The guarantee includes interest due on the debt, provided the interest rate is the economy's rate for default-free one-year nominal obligations.

The government enforces capital requirements to limit its exposure. At the beginning of a period, the consumer invests nominal debt and equity in the firm, which the firm uses to buy capital. The standard capital requirement limits debt to a fraction $1-\alpha$ of the value of the capital held. But there is an exception and an exception to the exception. The exception is that firms may have higher leverage if the debt they bring into a period exceeds the fraction $1-\alpha$ of the value of the capital. In that case they may keep debt at its previous level but may not take on any additional debt. The government forbears action against still-solvent firms that are in violation of the capital requirement. The exception to the exception is that debt may never exceed the value of the capital—the firm must be solvent during the period.

In principle, capital requirements for firms selling guaranteed debt often have the objective of disciplining firms that are solvent but lack all of the required capital, according to the doctrine of prompt corrective action (see Kocherlakota and Shim (2007) for an analysis of the doctrine that covers rather different issues from this paper). The obstacles to enforcing marking to market are serious, however. Asset valuations tend to use historical rather than market values. If the government pushes guaranteed borrowers to mark their portfolios to market, the borrowers shift to assets that defy reliable valuation. Financial institutions are remarkably willing and able to create these assets, as recent experience has shown.

When a firm becomes insolvent because a decline in the asset price lowers the value of capital below the value of debt, the government makes its payoff. The firm starts the next year without any legacy debt. Its new debt is constrained by the normal capital requirement.

The Modigliani-Miller property applies to any non-guaranteed debt the firm might issue. Such debt would have no effect on allocations in the model. For simplicity, I assume that firms issue only guaranteed debt.

In addition to capital requirements, I assume that the government can prevent specialization among firms that increases the government's exposure to payouts on debt guarantees. The danger is that some firms will pay out dividends to the point of borderline current solvency and that consumers, having reached the point where they prefer not to consume the dividends, invest them in another group of firms that are debt-free. I constrain all firms to have the same capital structures. The model would behave much the same way if specialization were permitted, but I exclude it to simplify the exposition.

The government finances the payments to honor its debt guarantees from a source other than current taxation. For example, it might have levied a one-time tax in the past and used the proceeds to purchase an insurance contract adequate to pay all of its guarantee obligations for the future. Modeling a current tax—even a lump-sum tax—would dramatically increase the complexity of the analysis without adding much to its substantive content.

3.4 Flows and returns

Debt pays a nominal interest rate of r_d. I discuss the determination of this rate in a later section. Because the government guarantee of debt is a giveaway, the consumer always lends the maximum permissible amount, which I write as

$$D = \min(p(1+r)K, \max(\hat{D}, (1-\alpha)p(1+r)K)). \tag{6}$$

Here D is the amount to be repaid, including interest. The nominal quantity pK is the value of the capital the firm will carry through the period, The outside min enforces the solvency requirement during the year, the exception to the exception. The nominal quantity $(1-\alpha)p(1+r)K$ is the standard capital requirement, but the max grants the exception for legacy shadow debt, $\hat{D}$. The consumer invests $D/(1+r_d)$ at the beginning of the year and receives D at the end of the year. Note that the capital requirement applies to the interest to be paid as well as the principal, but gives credit for the anticipated earnings of capital at rate r.

The consumer holds equity, Q, in the amount

$$Q = pK - \frac{D}{1+r_d} \tag{7}$$

at the beginning of the year, to make up the total assets of the firm, pK, at the beginning of the year. Unless the legacy exception applies, the equity investment is a fraction α of the total assets. The return the consumer earns on the equity investment is

$$\max(p'(1+r)K - D, 0). \tag{8}$$

This is the residual after payment of interest and principal.

3.5 Consolidating consumers and firms

To study the allocations in the model, I consolidate each consumer with the corresponding firm. The decisions the consumer makes directly as the manager of the consolidated entity are the same as those that would occur under any efficient alternative managerial arrangement that operated the firm on behalf of its shareholders. The allocations in the economy are the same when each firm is affiliated with one consumer as they would be if firms and consumers participated in capital and product markets, because of constant returns.

If the government did not guarantee debt, consumers would solve the dynamic program of equation (4). The debt guarantee gives the consumer an opportunity to capture additional value from the government payoff for default. The consolidated entity does not care about default itself, as it is a gain to the firm just offset by the loss to the consumer, but it does gain from the payment that the government makes when guaranteed debt defaults. Suppose first that the value of p' is enough to pay off all the debt and interest. Then the combined return to the consumer is $p'(1+r)K$, the sum of the face value of debt and interest, D, and the return to equity in equation (8). Now suppose that default occurs so that equity gets nothing and debt receives D in the form of the value of the capital $p'(1+r)K$ and a guarantee payment $G = D - p'(1+r)K$. The consumer's combined return is $p'(1+r)K + G$. The only substantive effect of the guarantee is to add value not otherwise attainable when the asset price falls enough to trigger default. The optimizing consumer arranges to capture this value by taking on as much debt as possible. The availability of the subsidy has striking effects on the consumer's incentives to save and consume.

3.6 Consumer decision-making

A consumer has two state variables, total nominal debt $\hat{D}$ and quantity of capital $\hat{K}$. At the beginning of a period, the consumer chooses a level of debt, D, to apply during the period and a level of consumption, c. The consumer always lends the maximum permissible amount in equation (6). The consumer invests $pK = p \cdot (\hat{K} - c)$ in the firm for the period.

The indicator variable z' takes the value one of default occurs next period and zero if not. The law of motion for capital is

$$\hat{K}' = (1 - z')(1 + r)K + z'\frac{D}{p'}. \tag{9}$$

Under solvency in the next period, the consumer earns the return r for the capital K held during the period. Under insolvency, the new quantity of capital is the purchasing power of the defaulted debt—the government makes up the difference between the actual amount of capital and the real value of the debt, so total capital is the real value of the debt.

The law of motion for debt is

$$\hat{D}' = (1 - z')D. \tag{10}$$

Debt at the beginning of a period is the amount held through the earlier period unless the consumer defaults, in which case it becomes zero.

As in the simple growth model, capital enters the value function as $\hat{K}^{1-\gamma}$. The consumer's dynamic program is

$$V\left(\frac{\hat{D}}{p\hat{K}}\right)\hat{K}^{1-\gamma} = \max_c \left(\frac{c^{1-\gamma}}{1-\gamma} + \mathbb{E}\,\frac{1}{1+r}V\left(\frac{\hat{D}'}{p'\hat{K}'}\right)\hat{K}'^{1-\gamma}\right). \tag{11}$$

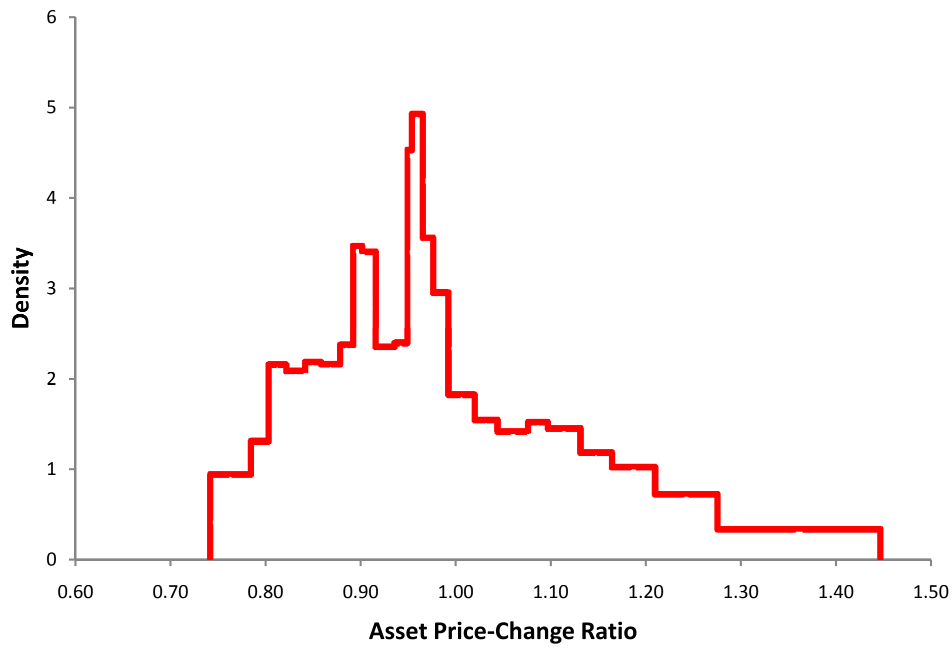

Figure 2: Density of Annual Price Change Ratio

4 Calibration

I use the standard value of $\gamma = 2$ for the coefficient of relative risk aversion. I take the annual net product of capital to be $r = 0.05$ and the capital requirement to be $\alpha =$ 30 percent. For the distribution of annual rates of change of asset prices, I use Robert Shiller's data for the S&P 500 and its predecessors over the period 1889 through 2004 (http://www.econ.yale.edu/∼shiller/data/chapt26.xls). I take the real quantity of capital underlying the price series to be proportional to consumption, so I take the price, p, to be the ratio of the S&P price to consumption and then calculate the price ratio as p'/p. I place the ratio into 25 bins of equal frequency to make the distribution discrete. Figure 2 shows the density of the distribution.

5 Equilibrium

The model implies that the capital stock evolves as a random walk. Positive shocks to wealth, arising from defaults, add to capital, while consumers deplete capital when they do not receive windfalls from the government. I will generally discuss the equilibrium of the model in terms of the ratio of consumption to the capital stock, $c/\hat{K}$, and the ratio of debt to the value of the capital stock, $\hat{D}/(p\hat{K})$.

At a time when legacy debt is low—just after default or following a positive price shock—the consumer will choose a value of debt controlled by the $(1-\alpha)p(1+r)K$ in equation (6). This choice is very close to 70 percent of the value of current capital $(1-\alpha)pK$ because consumption is chosen to be very close to rK. If the asset price rises in the next period, the consumer faces the same constraint and makes the same choice. If the asset price falls, the consumer becomes subject to the exception of keeping the old value of debt, $\hat{D}$. The ratio $\hat{D}/(p\hat{K})$, the consumer's debt-related state variable, is correspondingly higher. Because the consumer is closer to insolvency, the probability of default next period rises. This change triggers the key behavioral response of the model. The price of consumption is lower when the probability of default is higher. Writing out equation (11) as

$$V\left(\frac{\hat{D}}{p\hat{K}}\right)\hat{K}^{1-\gamma} = \max_c \frac{c^{1-\gamma}}{1-\gamma} + \mathbb{E}\,\frac{1}{1+r}\left\{(1-z')V\left(\frac{\hat{D}'}{p'\hat{K}'}\right)[(1+r)(\hat{K}-c)]^{1-\gamma} + z'V(0)\left(\frac{\hat{D}'}{p}\frac{p}{p'}\right)^{1-\gamma}\right\} \tag{12}$$

shows that when $z'=1$ and the consumer defaults, consumption turns out to have been free on the margin. The understanding of the possibility that consumption will turn out to be free results in a higher consumption choice before z' is realized. As random declines in the asset price push the consumer into higher values of $\hat{D}/(p\hat{K})$, consumption rises.

The consumer's Euler equation is instructive on this point. Suppose that neither the regular capital requirement nor the exception to the exception capital requirement is binding, so the exception permitting the carrying forward of legacy debt is in effect—see equation (6). Consider the standard variational argument where the consumer reduces current consumption by a small amount, saves that amount for one year, and then consumes that amount plus its earnings for the year. This variation has no implications for the level of debt. Let $c'(p'/p)$ be the level of consumption next year when the new price p' is known. Also let $F(p'/p)$ be the cdf of the price change. Then the Euler equation is

$$\int_{p^*}^{\infty} c'(p'/p)^{-\gamma} dF(p'/p) = c^{-\gamma}. \tag{13}$$

Here p^* is the value of the price ratio separating solvency from default. The Euler equation differs from the standard one (with non-stochastic return to capital equal to the rate of time preference) only in the truncation of the integral on the left side. The omission of part of the distribution of marginal utility on the left, for the cases where the bailout makes

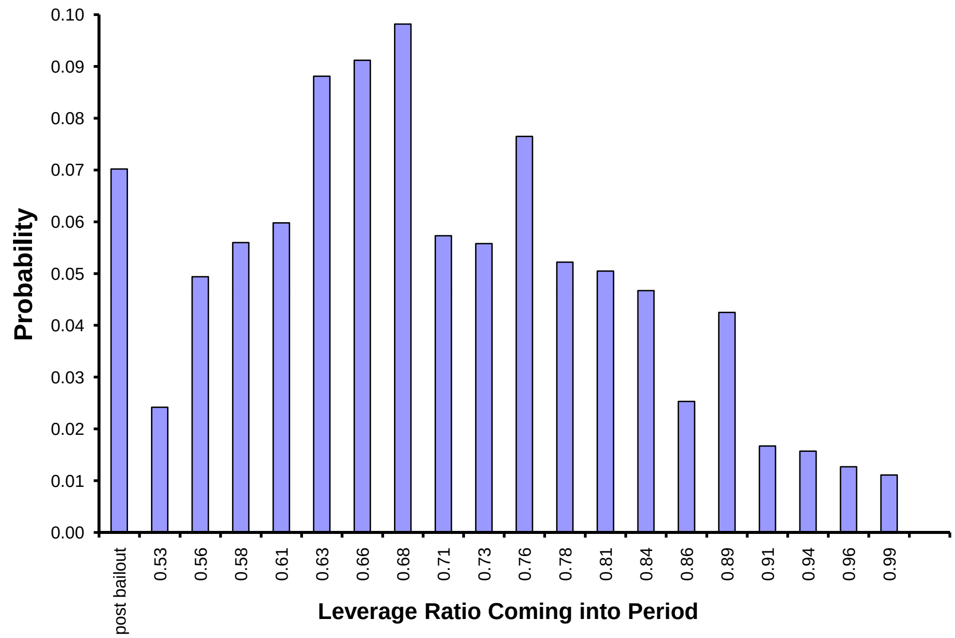

Figure 3: Distribution of Leverage Ratio

consumption free, results in a lower current marginal utility and thus a higher level of current consumption. The consumer chooses high current consumption and plans that consumption will fall in the next year, on the average.

The model implies a Markoff transition process for the financial position of the consumer as measured by $\hat{D}/(p\hat{K})$. Figure 3 shows the stationary distribution of the process. I approximate the process by a 200-state version and then aggregate to the bins shown in the figure. About half the time, $\hat{D}/(p\hat{K})$ is at or below the notional limit of 70 percent. Although consumers will move up to the 70-percent level in any period when $\hat{D}/(p\hat{K})$ falls below 70 percent, positive price shocks are common and large, so the state variable is often below 70 percent because the most recent shock was positive. Seven percent of the time, consumers consumers have no legacy debt because they defaulted in the previous period. For the other half of the time, consumers have higher leverage than 70 percent because the price level is below its previous peak.

Figure 4 shows the consumer's choice of debt for the coming period, D, which I normalize as $D/(p\hat{K})$. Much of the time the choice is $(1-\alpha)(1+r)K$, the 70 percent mentioned above. This is the choice of a consumer just out of the punishment state and of a consumer who is not eligible for the legacy exception. Whenever the asset price falls, the consumer is eligible

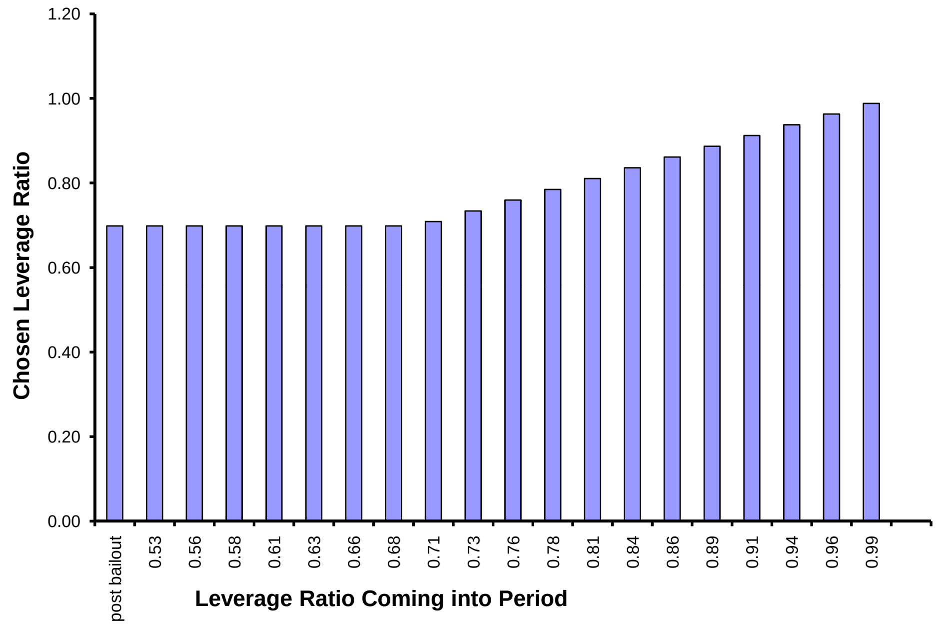

Figure 4: Chosen Leverage Ratio

for the legacy exception, either as a continuation and extension of the exception or for the first time. In years following multiple price declines, consumers may enjoy values of the leverage ratio $D/(p\hat{K})$ close to its upper limit of almost exactly one—the exception to the exception, where $D/(p\hat{K}) = (1 + r)(1 - c/\hat{K})$.

Figure 5 shows the probability of default conditional on the leverage ratio. It rises monotonically to its highest possible level of 36 percent. The discrete distribution of the price change has 9 values where the price declines and 16 where it rises. Each value has probability 0.04, so the probability of a price decline is $9 \times 0.04 = 0.36$.

Figure 6 shows how consumption responds to the state of the economy. In the region of the legacy exception ($0.703 \leq \hat{D}/(p\hat{K}) \leq 0.987$) consumption rises with the leverage ratio because the probability of default rises, making consumption this period cheaper because the government finances it when default occurs. At the highest value of the debt ratio, consumption is much lower, because of the exception to the exception requiring solvency during the period.

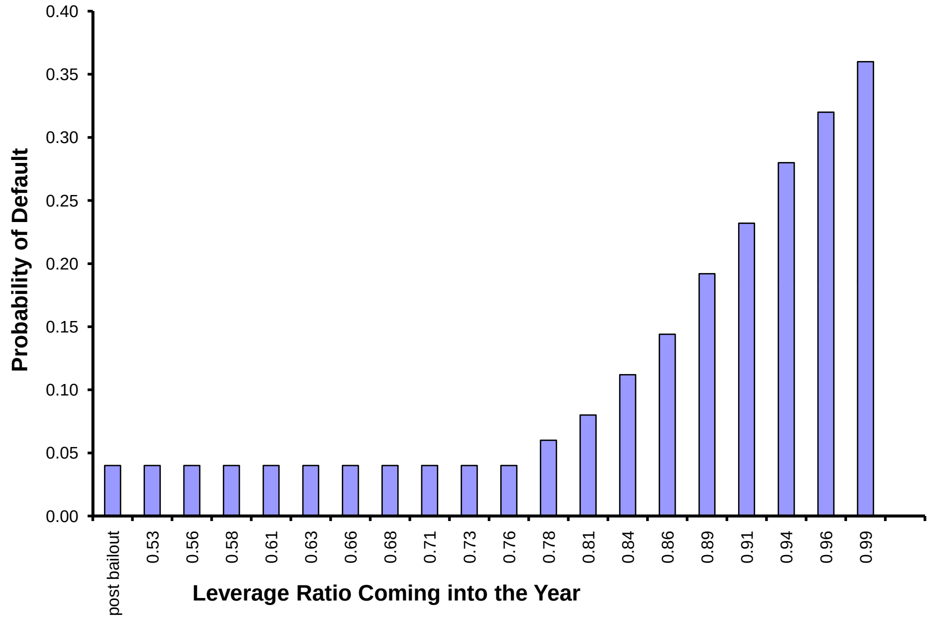

Figure 5: Probability of Default as a Function of the Leverage Ratio

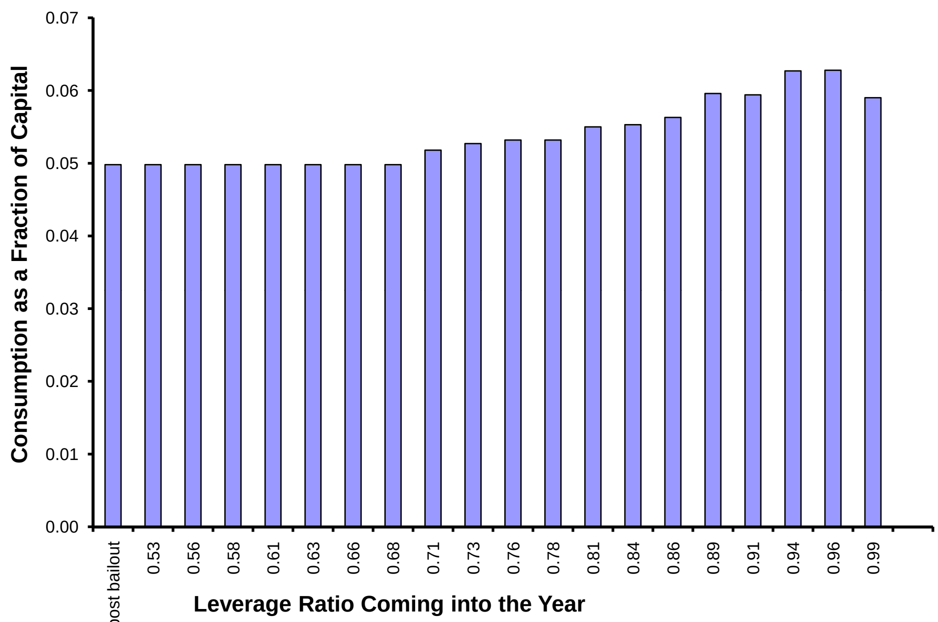

Figure 6: Consumption/Capital Ratio as a Function of the Leverage Ratio

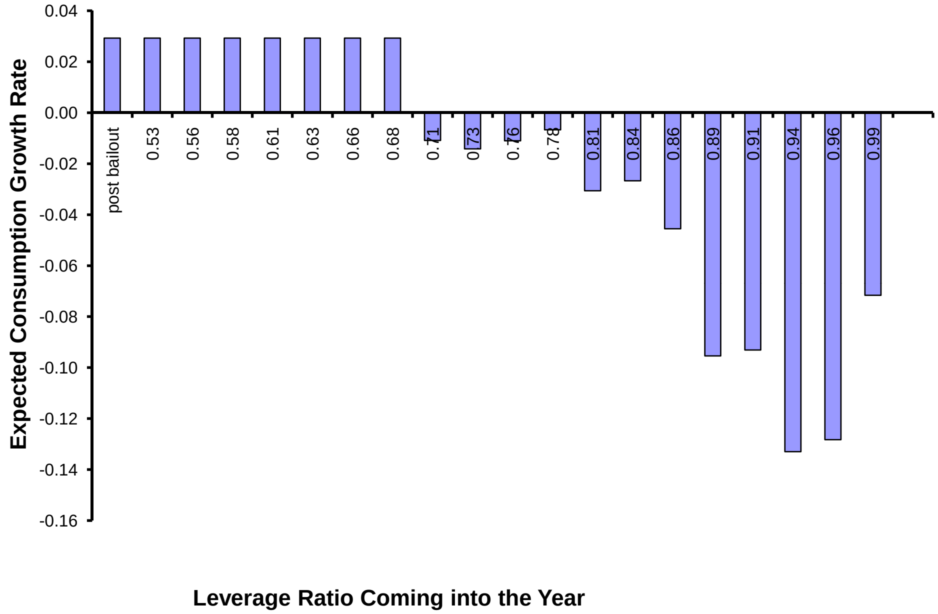

Figure 7: Expected Consumption Growth Rate as a Function of the Leverage Ratio

5.1 Nominal interest rate on debt

The nominal interest rate on debt, r_d, satisfies the consumption CAPM asset-pricing condition,

$$\frac{1+r_d}{1+r}\,\mathbb{E}\,\frac{p}{p'}\left(\frac{c'}{c}\right)^{-\gamma} = 1. \tag{14}$$

As usual in the consumption CAPM, the rate varies positively with expected consumption growth. It also varies because consumption growth covaries with $\frac{p}{p'}$ (the conditional distribution of $\frac{p}{p'}$ is invariant but the conditional distribution of $\frac{c'}{c}$ varies by current state). Figure 7 shows expected consumption growth as a function of the financial condition of the consumer. Consumption hardly changes at all when the consumer is in the post-default state, so the expected growth rate in that state is essentially zero. Consumers near or below their capital requirements expect consumption to grow, as there is a change that they will move next year into a more leveraged position where a higher level of consumption is optimal. But once consumers reach higher leverage, they face the high likelihood of default with a sharp drop of consumption back to the punishment or normal leverage level, so expected consumption growth is negative.

Figure 8 shows the nominal interest rate on guaranteed debt as a function of leverage. When leverage is low, the rate is 13 percent—5 percent real return to capital plus 4 percent

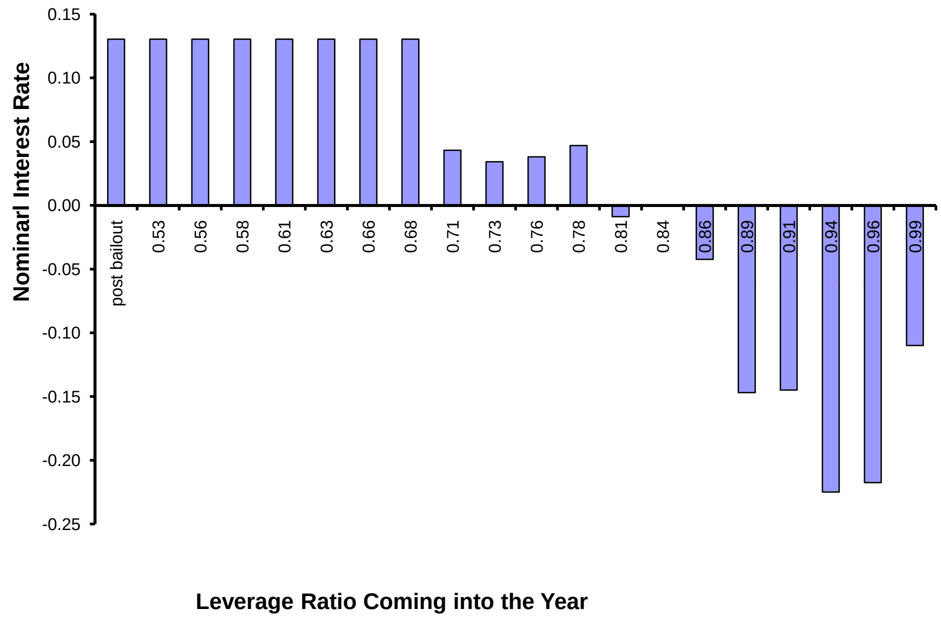

Figure 8: Nominal Interest Rate as a Function of the Leverage Ratio

expected inflation plus about 4 percent from expected consumption growth. As long as leverage does not exceed 79 percent, the rate remains above 3 percent. For higher amounts of leverage, the force of expected declines in consumption takes over. Above 80 percent leverage, the rate becomes negative as participants know that default and the accompanying resumption of the normal, lower level of consumption is fairly likely (see Figure 5). As formulated, the economy does not have a zero lower bound on the nominal rate.

5.2 Return to equity

The equity investment $Q = pK - D/(1 + r_d)$ in one period pays off

$$(1 + r_Q)Q = \max(p'(1 + r)K - D, 0). \tag{15}$$

The return satisfies the pricing condition,

$$\mathbb{E}\ \frac{1 + r_Q}{1 + r}\frac{p}{p'}\left(\frac{c'}{c}\right)^{-\gamma} = 1. \tag{16}$$

5.3 Financial flows

In one year, after consuming the amount c, the owner lends the firm $D/(1+r_d)$ and provides equity Q which adds up to capital worth pK in the hands of the firm. Production rK occurs before the next year and a new price realization p' occurs. The firm now has capital worth $p'(1+r)K$. It pays this amount to its owner as the return of borrowed funds, interest on those funds at rate r_d, and the payout of the owner's residual equity, if the firm has not defaulted. The government makes up the difference to the owner if $p'(1+r)K$ falls short of D. The model generates matrices of flows, indexed by the state of the economy in the first year, $\hat{D}/(p\hat{K})$, and by the price realization p' in the next year. I describe these matrices in two ways.

First, Figure 9 shows the expectation of the ratio of the flows to the original value of capital, $p\hat{K}$. The horizontal axis shows $\hat{D}/(p\hat{K})$, the amount of debt brought into the first period, as a ratio to the value of capital brought into the period. If the inherited leverage ratio is less than the notional upper limit of $1-\alpha$, 70 percent, the consumer lends the firm 70 percent of the value that the capital will have at the end of the period, at the earlier price, $(1-\alpha)p\cdot(\hat{K}-c)$. Default has a low probability, so the firm is able to repay the debt with interest, as shown in the top line, and to return the consumer's equity at a level with a reasonable return. The expected government guarantee payment, shown at the bottom, is only slightly positive (default occurs only if the asset price falls by its largest possible amount). The firm issues new debt and new equity with values somewhat below the amounts returned, reflecting the interest and return to equity from the earlier investments. The expected net outflow, shown close to the bottom, is the expected nominal value of the consumer's consumption at the beginning of the new period.

On the right side of Figure 9, the consumer benefits from the exception for legacy debt. Earlier declines in asset prices have left the consumer with the right to issue more debt to replace the high level of legacy debt. The rising lines show the expected repayment of debt including interest and the expected issuance of new debt. Their upward slope reflects the higher value of legacy debt. The bottom line shows the government's expected payoff. Expected return of equity and issuance of new equity decline with leverage (the irregularities in new debt and equity are the result of the discrete distribution of asset price changes). Equity depletion is visible as the excess of expected new debt over repayment of debt and the shortfall of new equity over the return of equity. Again, the expected net outflow is the amount needed to finance expected nominal consumption, which is essentially flat—the covariance of real consumption and the price level falls sufficiently with leverage to offset the rise in real consumption.

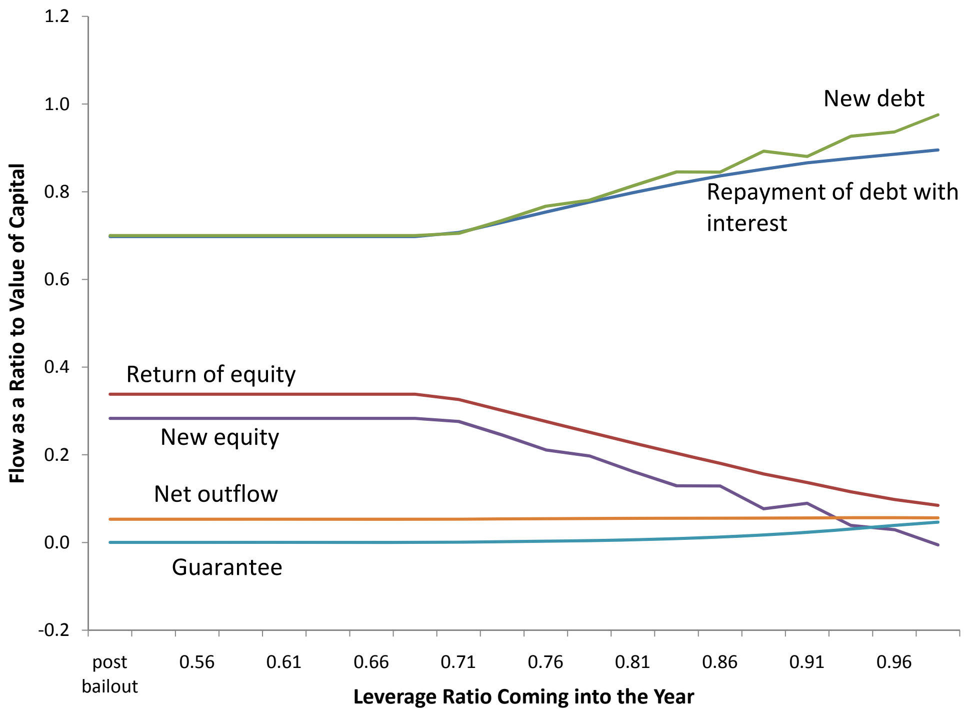

Figure 9: Expected Flows as Functions of the Leverage Ratio

The second view of the matrix of flows is along one row, showing the various possible outcomes at a given level of prior leverage. Figure 10 plots the flows with the price change on the horizontal axis and the flows as fractions of the earlier value of capital on the vertical axis, for $\hat{D}/(p\hat{K}) = 0.85$. For consistency with the other plots in the paper, where declines in asset prices move the economy to the right, the horizontal axis has a high new price on the left and a low new price on the right. The left side of the figure shows what happens when the economy retreats from high leverage because the asset price rises. No default occurs, so all debt is repaid with interest. Newly issued debt is about equal to debt repayments. For modest asset-price declines, the interest rate turns negative because of the impending decline in consumption—see Figure 8. Now an obligation to repay principal and interest at the end of the year yields more than a dollar of current funds for each dollar returned at the end of the year. New debt issuance rises to a peak, offset by a reduction in equity. Firms are borrowing heavily and paying out the proceeds as dividends or share repurchases. Dollar spending on consumption remains constant, but real spending rises, as shown in Figure 6. Farther to the right of Figure 10, default occurs. Repayment of debt drops because the firm's collateral has fallen below the face-value of debt and interest.The guarantee payment rises. New debt is at the same level as on the left, because the firm loses its legacy right to issue

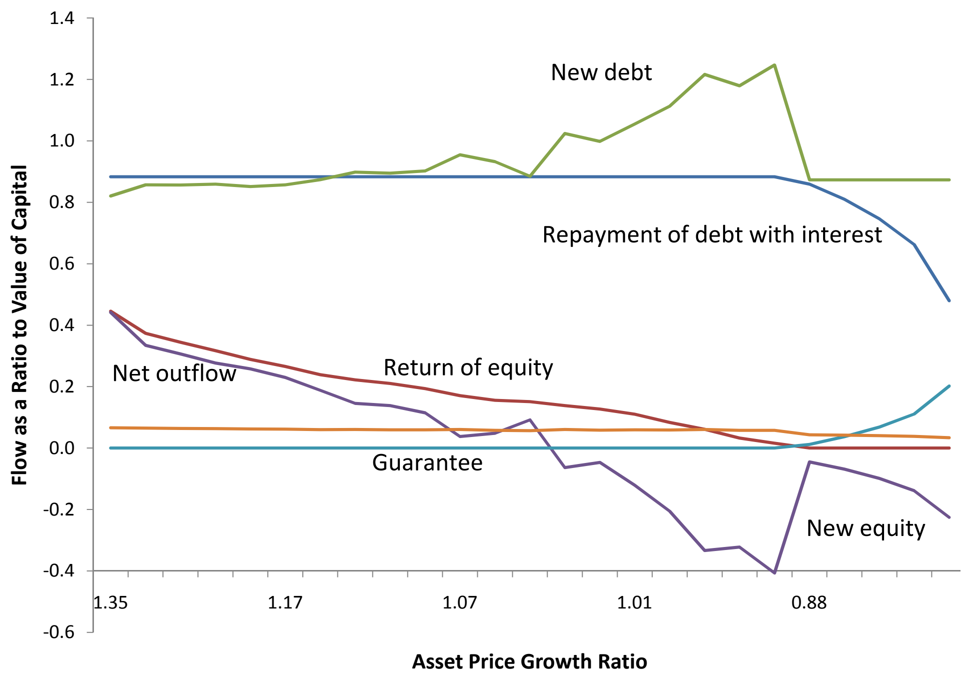

Figure 10: Flows as Functions of the Price Ratio when Prior Leverage is 0.85

more debt upon default. Issuance of equity makes up the difference.

5.4 Equity depletion

Equity depletion occurs when a decline in the asset price causes the probability of default to rise. Figure 10 shows how depletion occurs. If the asset price falls, the firm pays out a substantial amount of equity. Most is replaced by corresponding debt investment. Two factors account for the equity outflow that is replaced by debt. First is the compelling advantage of leverage when the government guarantees debt. From the joint perspective of the firm and investor, debt is unambiguously a good thing and the firm is always at a corner solution with maximum permissible guaranteed debt. A fall in the asset price relaxes the debt constraint because it lowers the interest rate. The firm immediately borrows up to the new, higher limit. Recall that the amount borrowed is $D/(1 + r_d)$. The firm then pays out the difference as dividends and thereby reduces equity. The second factor is that investors reduce their total funding when the asset price falls, because they increase consumption. The new level of funding to the firm is $p'(\hat{K}' - c')$ and c' rises when p' is below p. Although the decline in total funds available is small relative to the shift from equity to debt, the increase in consumption is critical to the process, because the rise in consumption as firms approach

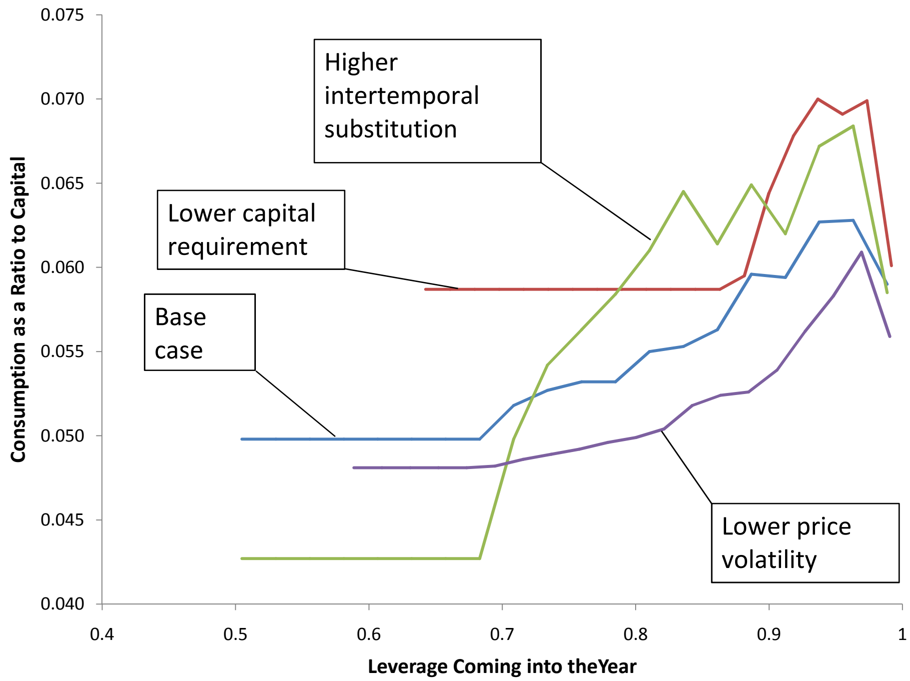

Figure 11: Comparison of Consumption as a Function of Leverage for Four Cases

default creates a decrease in expected consumption growth and thus the lower interest rate that permits the expansion of debt.

6 Roles of Key Parameters

Figure 11 compares economies with different values of the key parameters, in terms of the way consumption depends on the leverage state variable, $\hat{D}/(p\hat{K})$. The first case is a capital requirement of $\alpha = 0.1$ instead of 0.3 as in the base case. Firms coming into the year with leverage less than 90 percent immediately issue debt up to that point. Consumption is at the same level for all amounts of beginning leverage below 90 percent, but a higher level than in the economy of the base case, because consumers anticipate a higher expected level of subsidy from the episodic guarantee payments they will receive at random times in the future. As in the base case, consumption rises to even higher levels when the guarantee payments become imminent, as the fraction of current consumption that will be charged to the consumer drops well below one.

The second case is less risk aversion and higher intertemporal substitution—specifically, a value of the curvature parameter γ of 0.5 in place of its value of 2 in the base case. Higher substitution increases the volatility of consumption. Consumers concentrate their spending in times of cheap consumption to a greater extent, so consumption is lower when leverage is at or below the capital requirement and higher when leverage is higher and the probability of default is higher, so consumption is cheaper in expectation.

The third case is lower volatility of the asset price. I shrink the distribution of the price-change ratio, p'/p, toward 1 by 50 percent. The result is lower consumption than in the base case at all levels of leverage. Consumption is lower at low leverage because consumers have lower lifetime incomes when guarantee payments are smaller on the average. Consumption rises by less when the consumer's firm enters the region of higher leverage because the probability of default is lower at any given amount of leverage if large price declines are less likely.

Volatility in other variables follows closely from the volatility in consumption shown in Figure 11. In particular, movements in the interest rate track expected consumption growth, which is more volatile when intertemporal substitution is higher (γ is lower), but less volatile with a lower capital requirement and with lower asset-price volatility.

7 Concluding Remarks

The essential feature of a government guarantee of debt that yields the results in this paper is the government's failure to take prompt corrective action. Asset-price declines permit higher leverage, with a higher probability of government payoff. The consumption bulge that is the source of the volatility would not occur if the government insisted on equity contributions to make up for price declines so as to keep leverage constant. Thus the relevance of the basic mechanism studied in this paper to modern economic instability rests on the government's failure to measure the market value of the collateral backing guaranteed debt and the government's resulting failure to require equity infusions in times of asset-price deflation. My impression is that these failures are the rule. Many organizations enjoying effective guarantees, such as investment banks, do not even have stated capital requirements. Financial intermediaries have shown great ingenuity in creating instruments that defy current valuation.

Government guarantees are only one of many issues raised by recent financial events. The point of this paper is to pursue the effects of guarantees in a dynamic general-equilibrium setting. Guarantees introduce volatility in key variables in an economy that would otherwise evolve smoothly. In particular, if the government administers guarantees in terms of nominal measures, then otherwise neutral nominal developments have profound real effects.

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